i s t a n b u l **t** e c h n i c a l **u** n i v e r s i t y
architectural **d**esign **c**omputing **g**raduate **p**rogram
dads 2011: a springtime venture

The Digital Shape or...
Mind the Gap Reloaded!

Lecture VIII
Non-**U**niform **R**ational **B**-**S**plines (NURBS)

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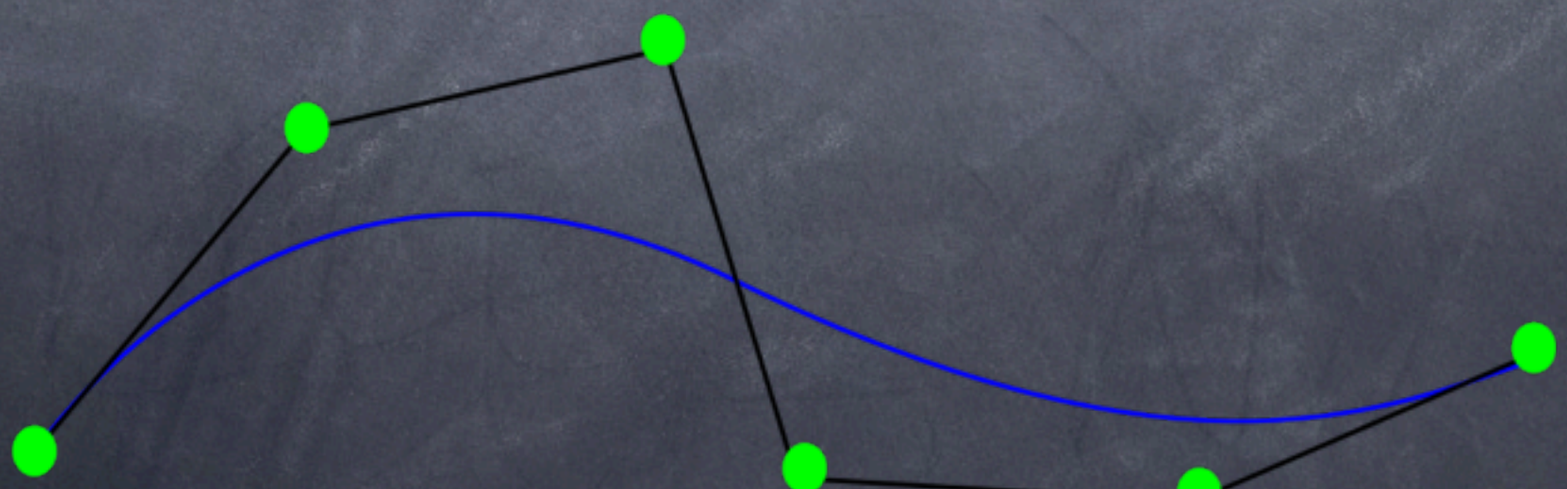
What are NURBS?

- Mathematical representations of 3D geometry: **parametric curves or surfaces**
- Accurately describe any shape from a simple 2D line, circle, arc, or curve to the most complex organic free-form 3D surface
- A NURBS curve is obtained by the superposition of parametric basis curves (splines) around control points off that curve.



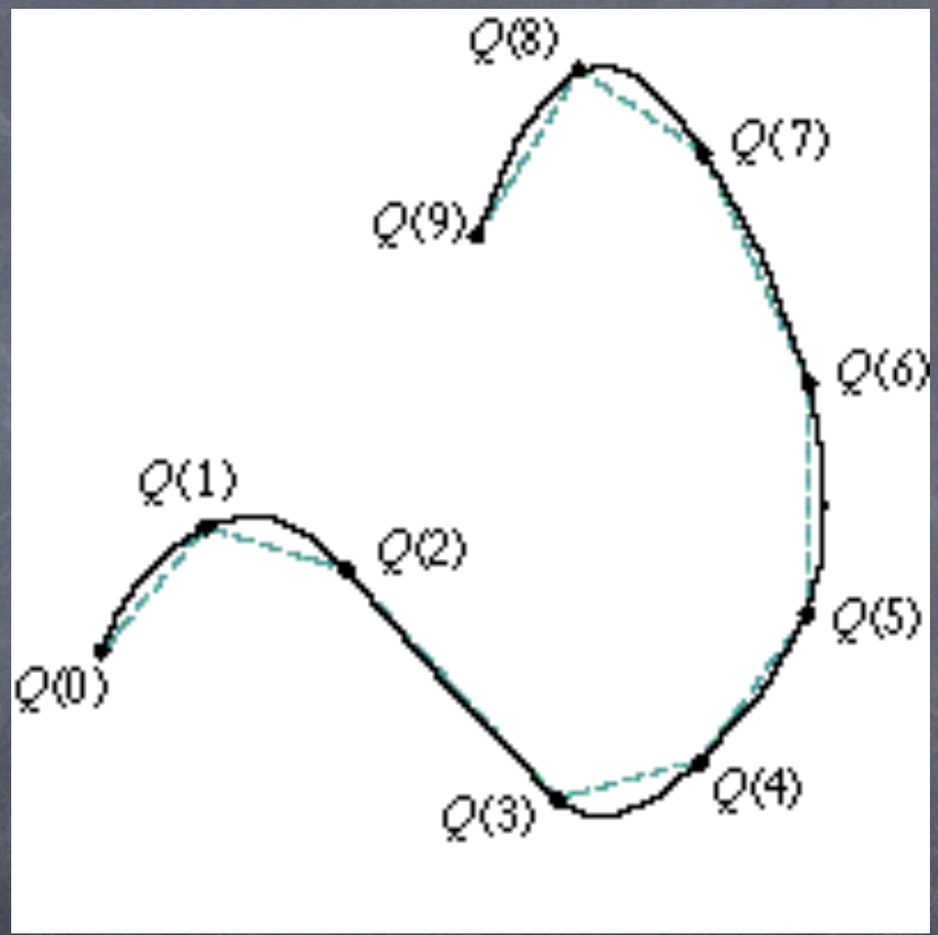
Why are NURBS so good?

- **Exchangeable.** Users can move their valuable geometric models between various modeling, rendering, animation, and engineering software programs
- **Clear and precise** mathematical definition
- **Flexible.** Can describe arbitrary free-form shapes
- **Parsimony.** Can be stored very efficiently
- **Programmatically simple and accurate**

NURBS Terminology

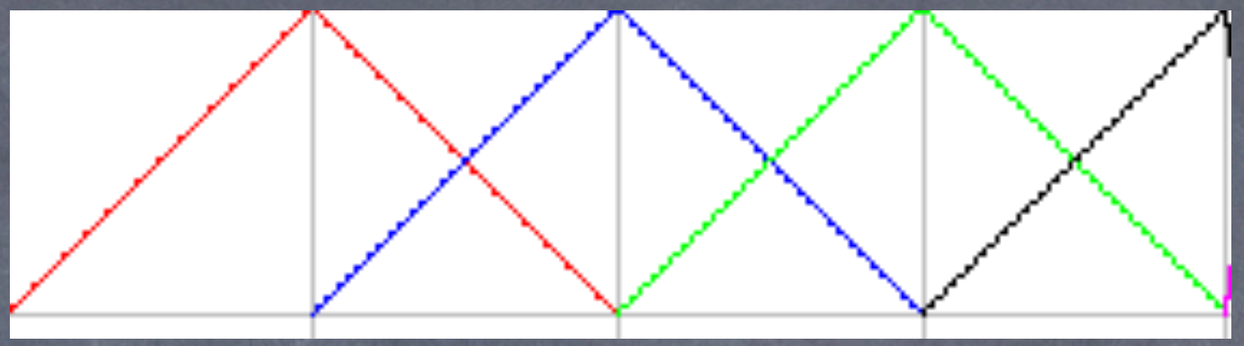
- **Degree (d).** Mathematical degree of the parametric basis function used, 1 (linear) for line, 2 (quadratic) for parabola or circle, 3 (cubic) or 5 (quintic) for arbitrary curves.
- **Control point (N).** At least d+1 geometrical points lying off the curve. The curve is locally pulled to or from a control point.
- **Knot.** d+N-1 points partitioning the parameter interval. The knot spacing determines the validity of a basis function around a certain control point.
- **Evaluation rule.** A mathematical formula involving degree, control points and knots. It takes a number (a specific parameter value) and yields a geometrical point.

Parametric Curve

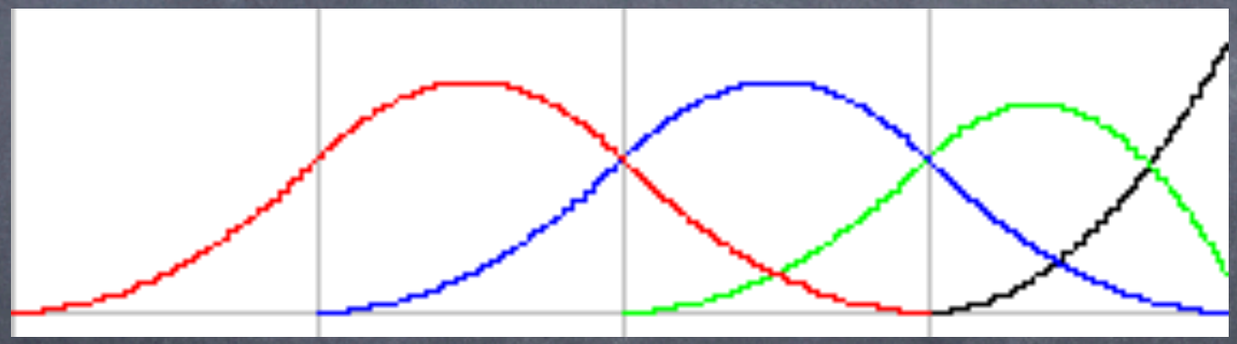


Degree

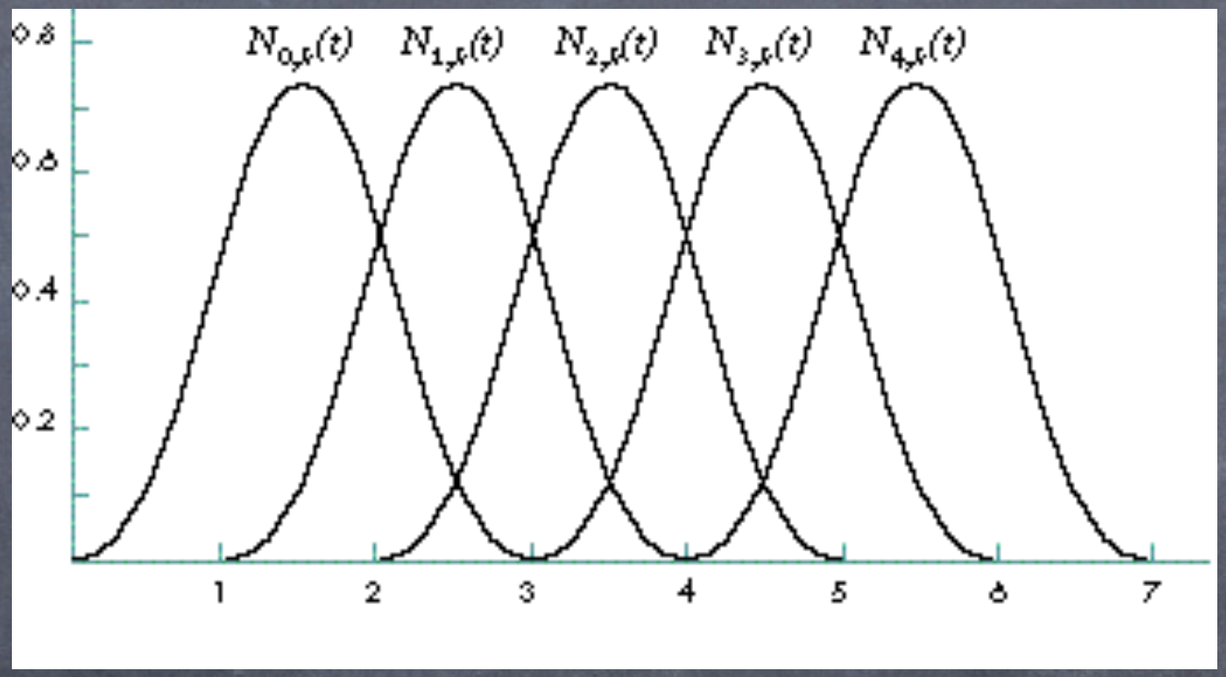
Linear



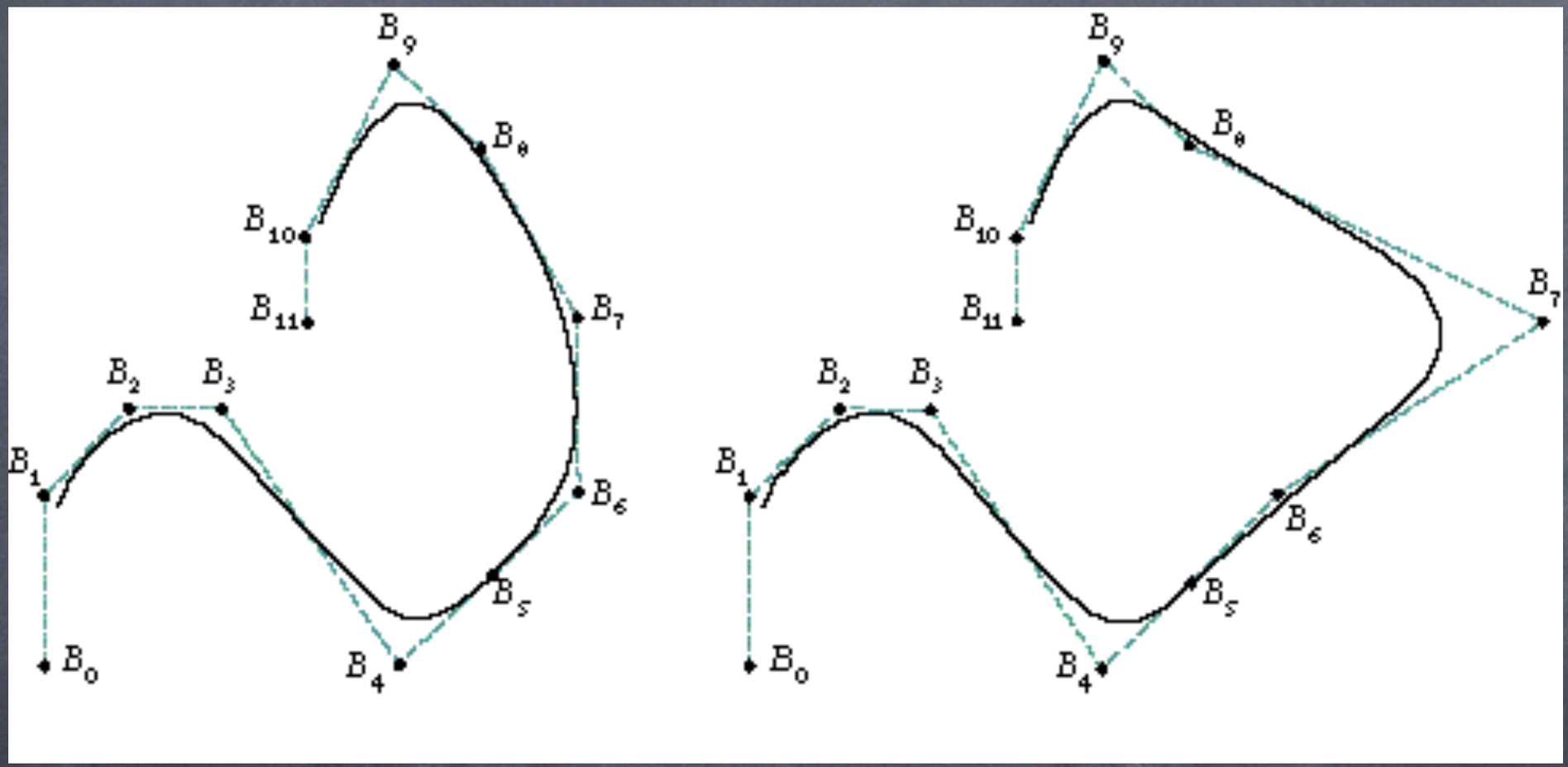
Quadratic



Basis Functions

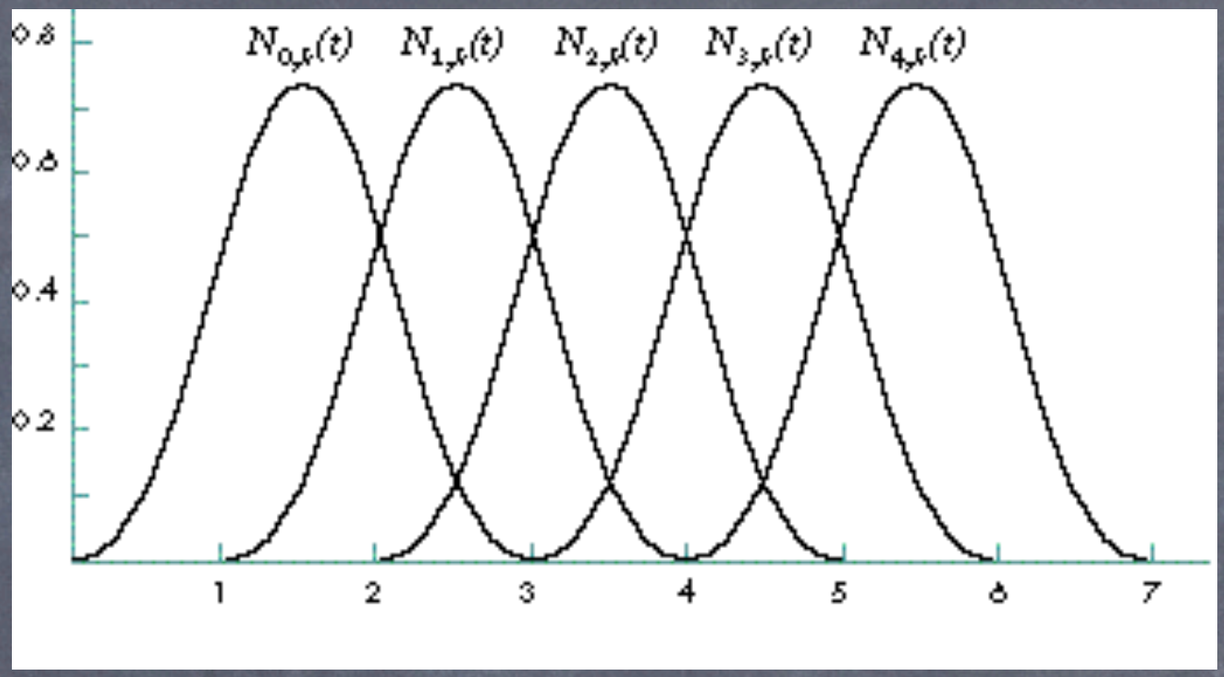


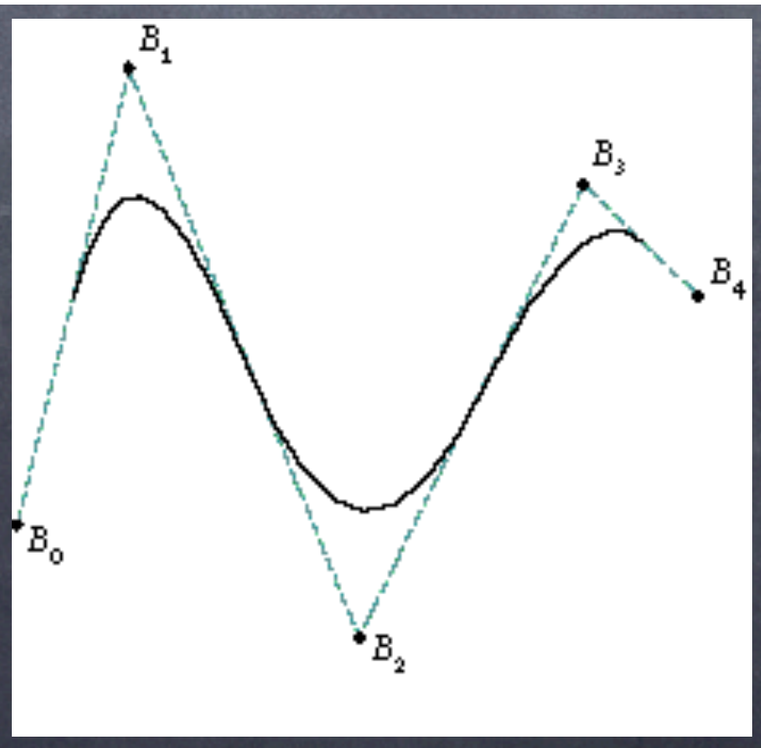
Control Points



Knots: Uniformly Spaced

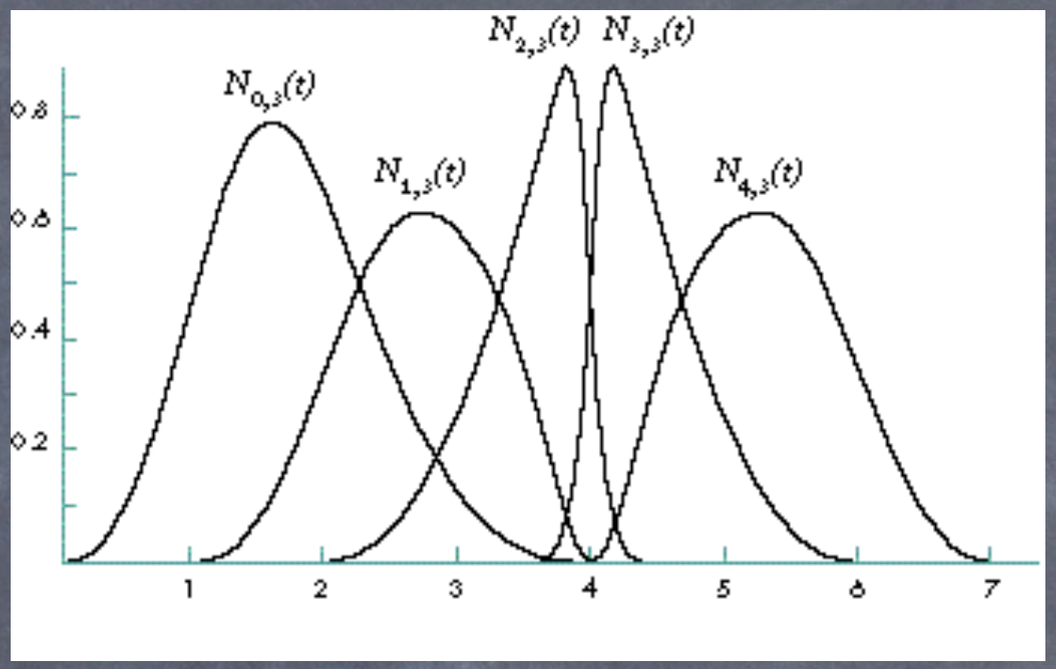
{0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0}

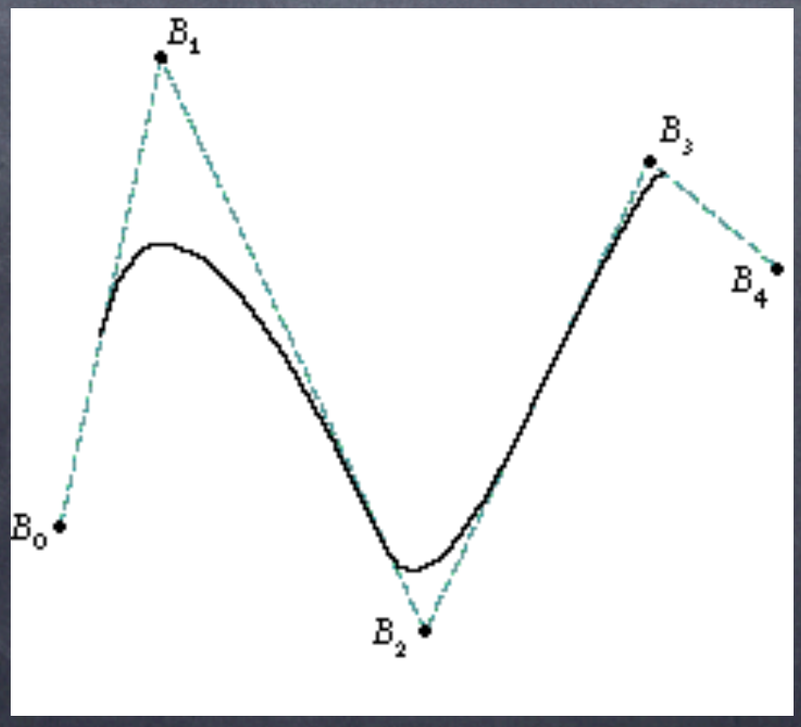




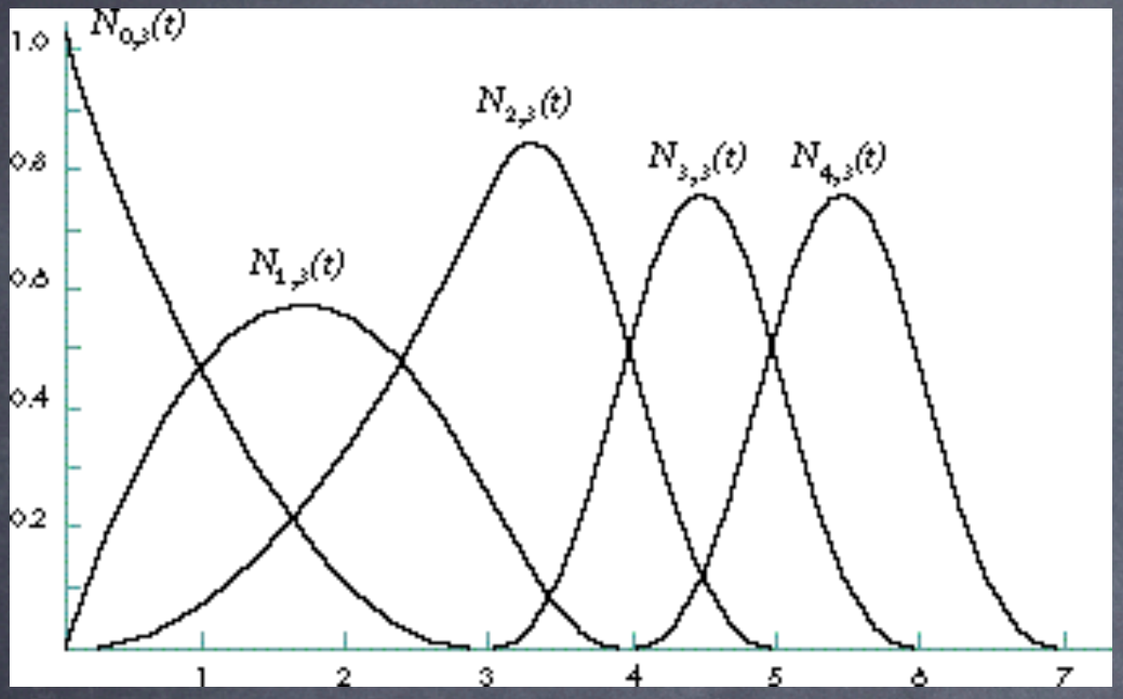
Knots: Non-Uniformly Spaced

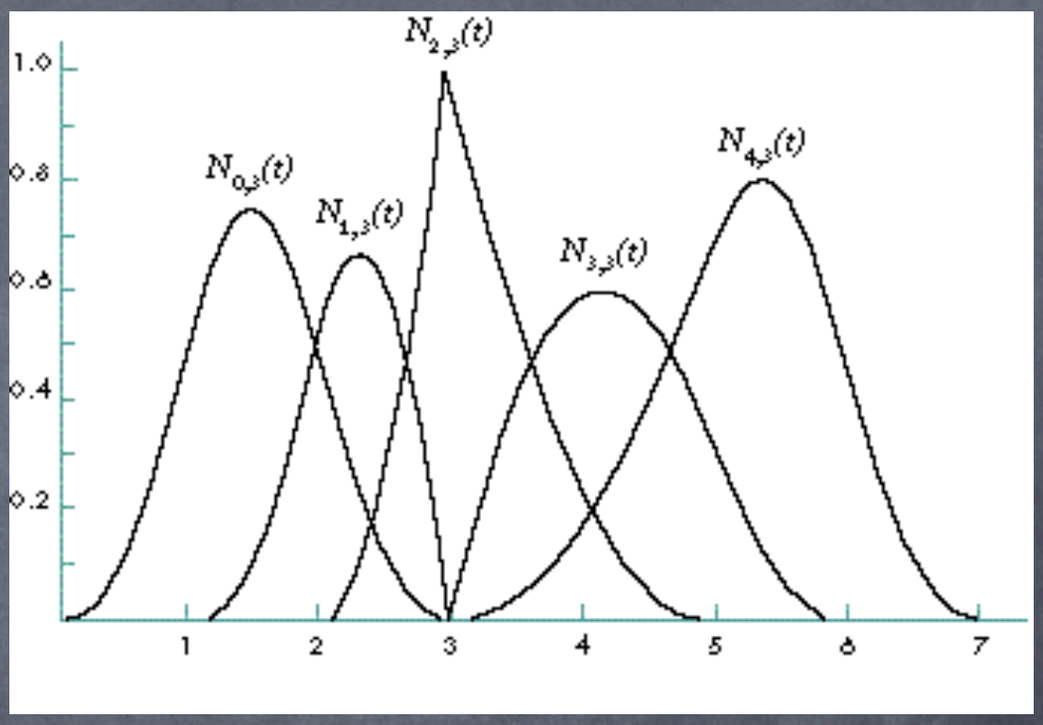
{0.0, 1.0, 2.0, 3.75, 4.0, 4.25, 6.0, 7.0}

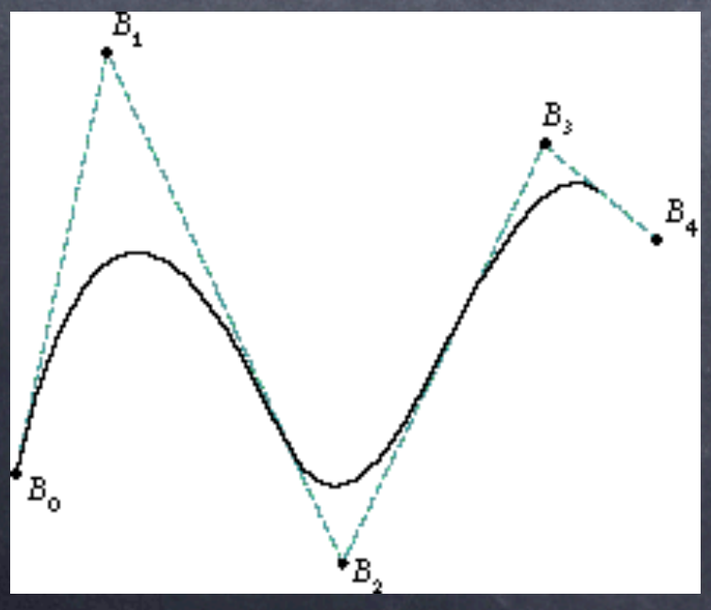


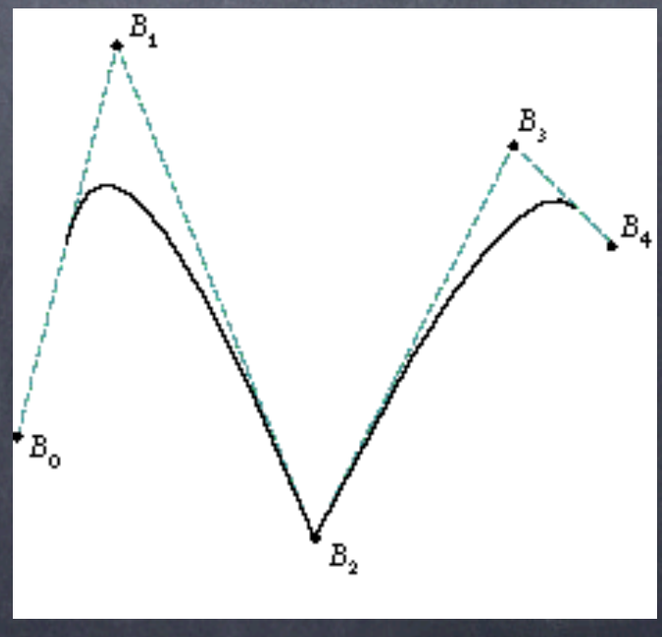


Repeating a Knot (Multiplicity)









Things to know...

- A point on a NURBS curve is obtained by a combination of weighted control points
- The weights are given by basis functions
- Knot spacings allow to create a virtually unlimited number of basis function configurations
- Repeating knots allows to incorporate kinks to a NURBS representation
- You can give extra privilege to a control point by weighting it more (apart from the weighting enabled by the basis function)

Some math facts...

A basis function of degree 0 is piece-wise constant.
It gets the value 1 at its corresponding knot.

Basis functions of higher degree are obtained recursively from basis functions of lower order

$$N_{i,n} = f_{i,n} N_{i,n-1} + g_{i+1,n} N_{i+1,n-1}$$

$$f_{i,n}(u) = \frac{u - k_i}{k_{i+n} - k_i}$$

and

$$g_{i,n}(u) = \frac{k_{i+n} - u}{k_{i+n} - k_i}$$