Computer Vision Course
Lecture 08

Feature Matching

Ceyhun Burak Akgül, PhD
cba-research.com

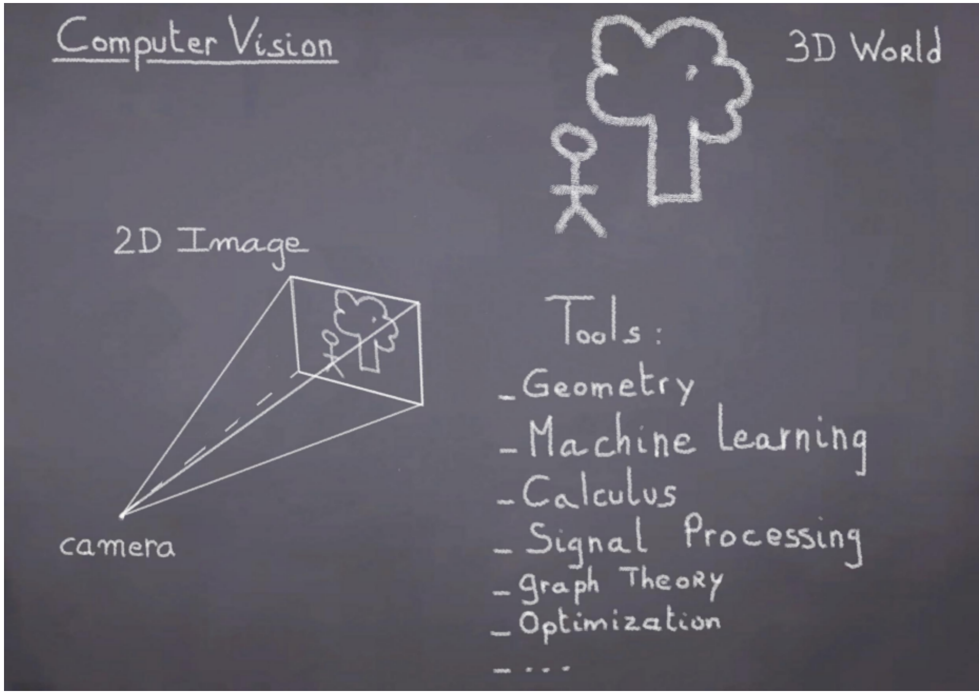


Photo credit: Olivier Teboul
vision.mas.ecp.fr/Personnel/teboul

Spring 2015
Last updated 22/04/2015

These slides have been adapted from James Hays's 2014 Computer Vision course slides at Brown University.

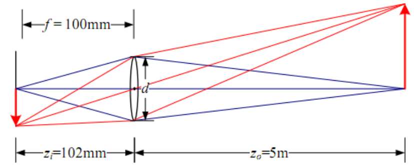
Course Outline

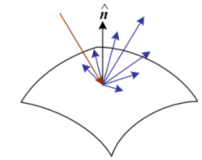
Image Formation and Processing

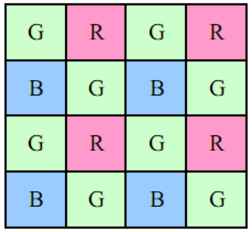
Light, Shape and Color

The Pin-hole Camera Model, The Digital Camera

Linear filtering, Template Matching, Image Pyramids







Feature Detection and Matching

Edge Detection, Interest Points: Corners and Blobs

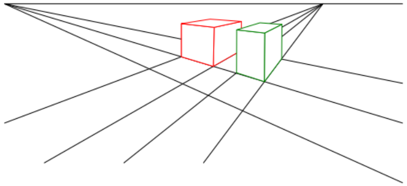
Local Image Descriptors

Feature Matching and Hough Transform

Multiple Views and Motion

Geometric Transformations, Camera Calibration

Feature Tracking , Stereo Vision



Segmentation and Grouping

Segmentation by Clustering, Region Merging and Growing

Advanced Methods Overview: Active Contours, Level-Sets, Graph-Theoretic Methods

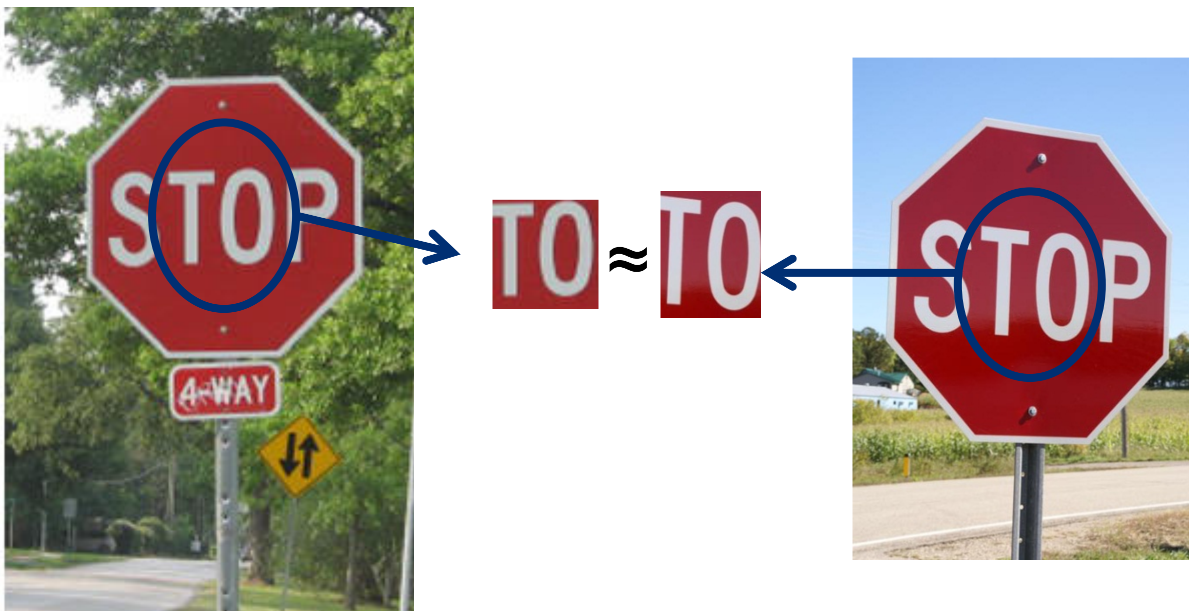
Detection and Recognition

Problems and Architectures Overview

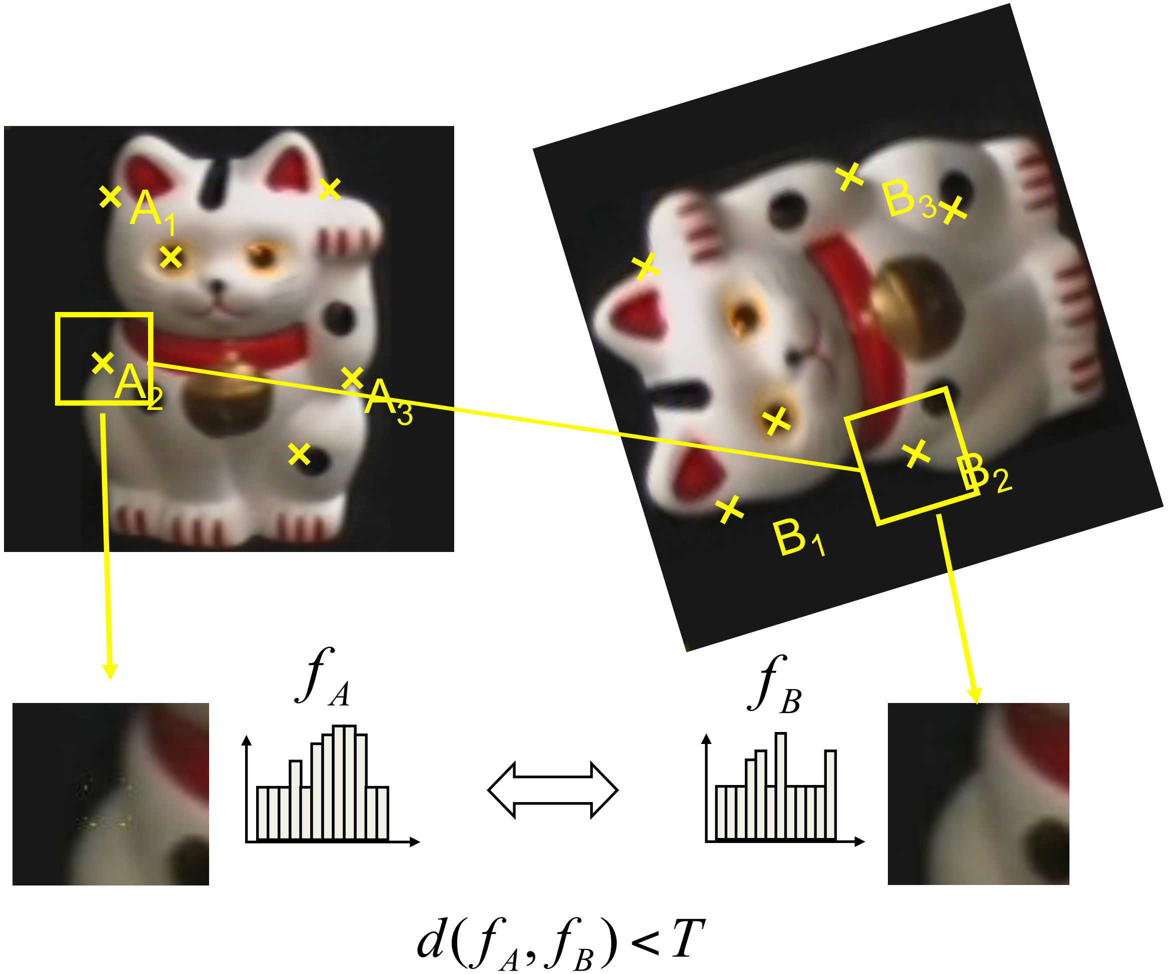
Statistical Classifiers, Bag-of-Words Model, Detection by Sliding Windows

Correspondence and Alignment

- Correspondence: matching points, patches, edges, or regions across images



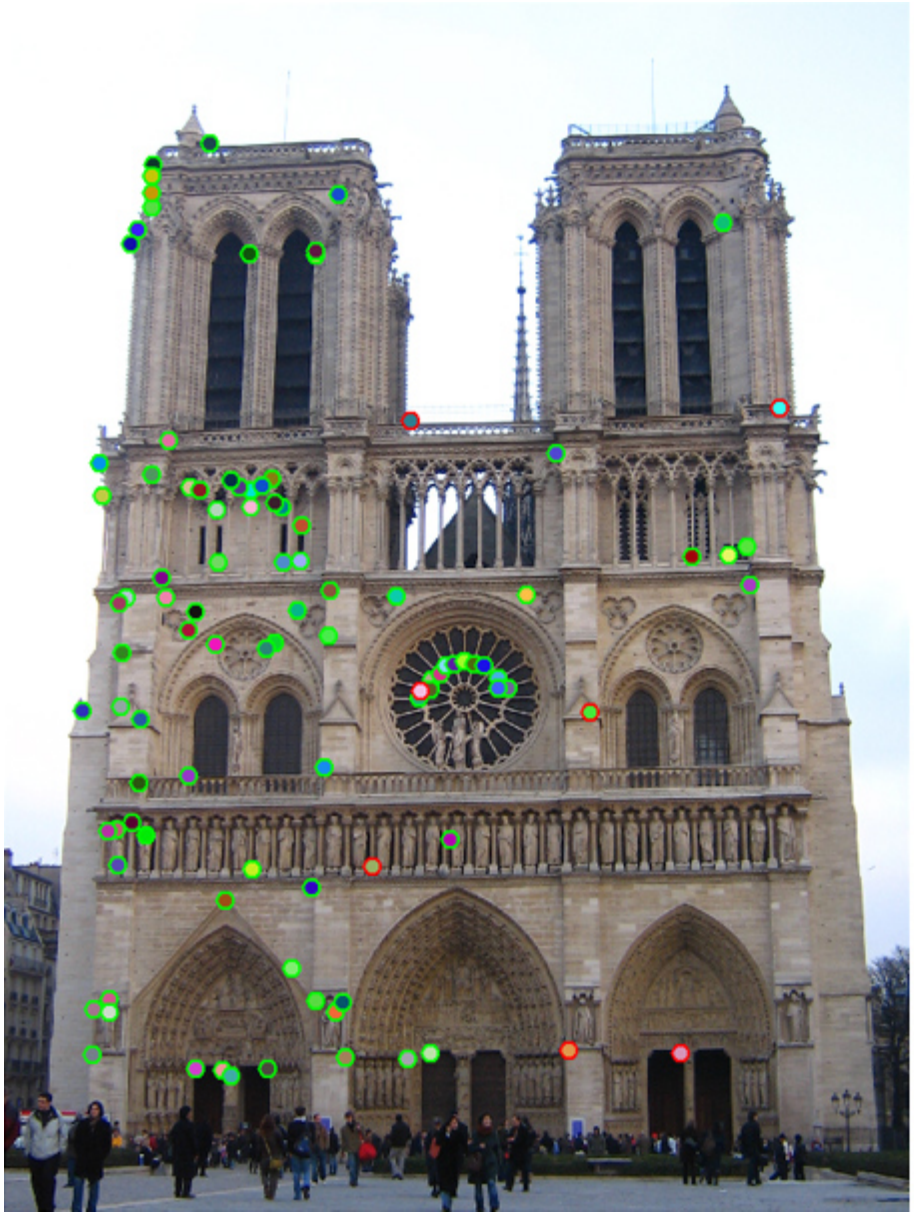
Overview of Keypoint Matching

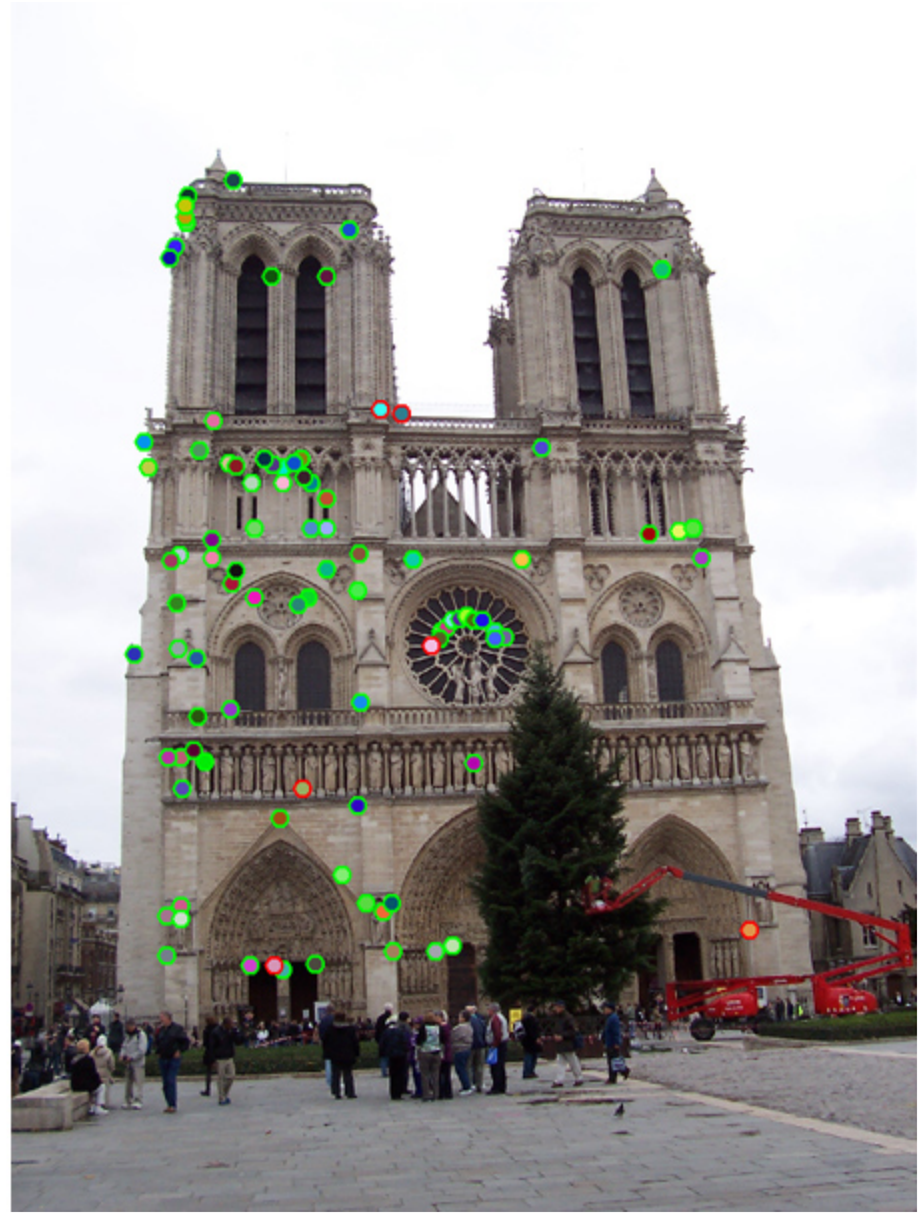


1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

K. Grauman, B. Leibe

How do we decide which features match?



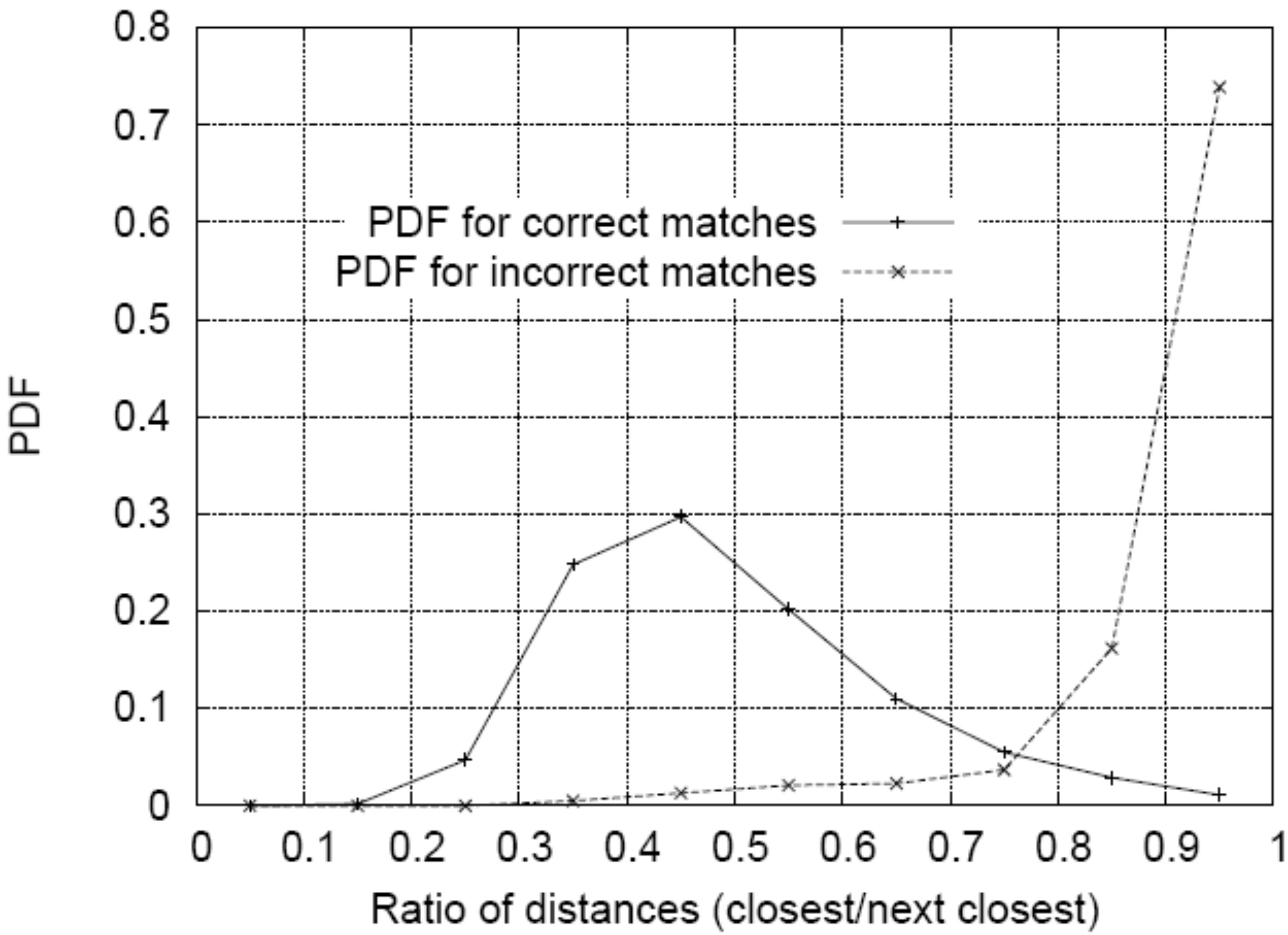


Feature Matching

- Simple criteria: One feature matches to another if those features are nearest neighbors and their distance is below some threshold.
- Problems:
  - Threshold is difficult to set
  - Non-distinctive features could have lots of close matches, only one of which is correct

Matching Local Features

- Threshold based on the ratio of 1st nearest neighbor to 2nd nearest neighbor distance.



Lowe IJCV 2004

Fitting and Alignment

Fitting: find the parameters of a model that best fit the data

Alignment: find the parameters of the transformation that best align matched points

Fitting and Alignment

- Design challenges
  - Design a suitable **goodness of fit** measure
    - Similarity should reflect application goals
    - Encode robustness to outliers and noise
  - Design an **optimization** method
    - Avoid local optima
    - Find best parameters quickly

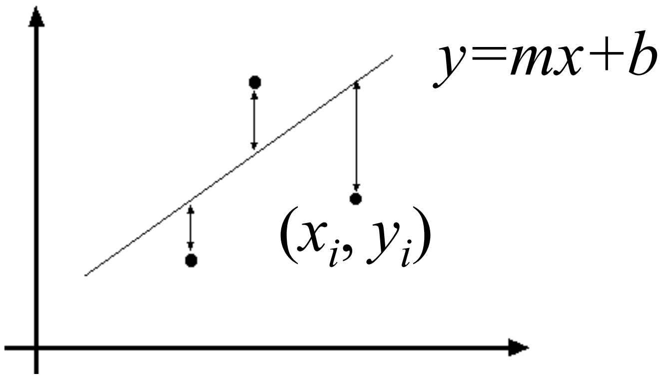
Fitting and Alignment: Methods

- Global optimization / Search for parameters
  - Least squares fit
  - Robust least squares
  - Iterative closest point (ICP)

- Hypothesize and test
  - Generalized Hough transform
  - RANSAC

Least squares line fitting

- Data: $(x_1, y_1), \ldots, (x_n, y_n)$
- Line equation: $y_i = m x_i + b$
- Find (m, b) to minimize



$$E = \sum_{i=1}^{n} (y_i - mx_i - b)^2$$

$$E = \sum_{i=1}^{n} \left(\begin{bmatrix} x_i & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - y_i \right)^2 = \left\| \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} - \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \right\|^2 = \|\mathbf{Ap} - \mathbf{y}\|^2$$

$$= \mathbf{y}^T \mathbf{y} - 2(\mathbf{Ap})^T \mathbf{y} + (\mathbf{Ap})^T (\mathbf{Ap})$$

$$\frac{dE}{dp} = 2\mathbf{A}^T \mathbf{Ap} - 2\mathbf{A}^T \mathbf{y} = 0$$

Matlab: `p = A \ y;`

$$\mathbf{A}^T \mathbf{Ap} = \mathbf{A}^T \mathbf{y} \Rightarrow \mathbf{p} = \left(\mathbf{A}^T \mathbf{A}\right)^{-1} \mathbf{A}^T \mathbf{y}$$

Modified from S. Lazebnik

Least squares (global) optimization

Good

- Clearly specified objective
- Optimization is easy

Bad

- May not be what you want to optimize
- Sensitive to outliers
  - Bad matches, extra points
- Doesn't allow you to get multiple good fits
  - Detecting multiple objects, lines, etc.

Hypothesize and test

1. Propose parameters
   – Try all possible
   – Each point votes for all consistent parameters
   – Repeatedly sample enough points to solve for parameters

2. Score the given parameters
   – Number of consistent points, possibly weighted by distance

3. Choose from among the set of parameters
   – Global or local maximum of scores

4. Possibly refine parameters using inliers

Hough Transform: Outline

1. Create a grid of parameter values

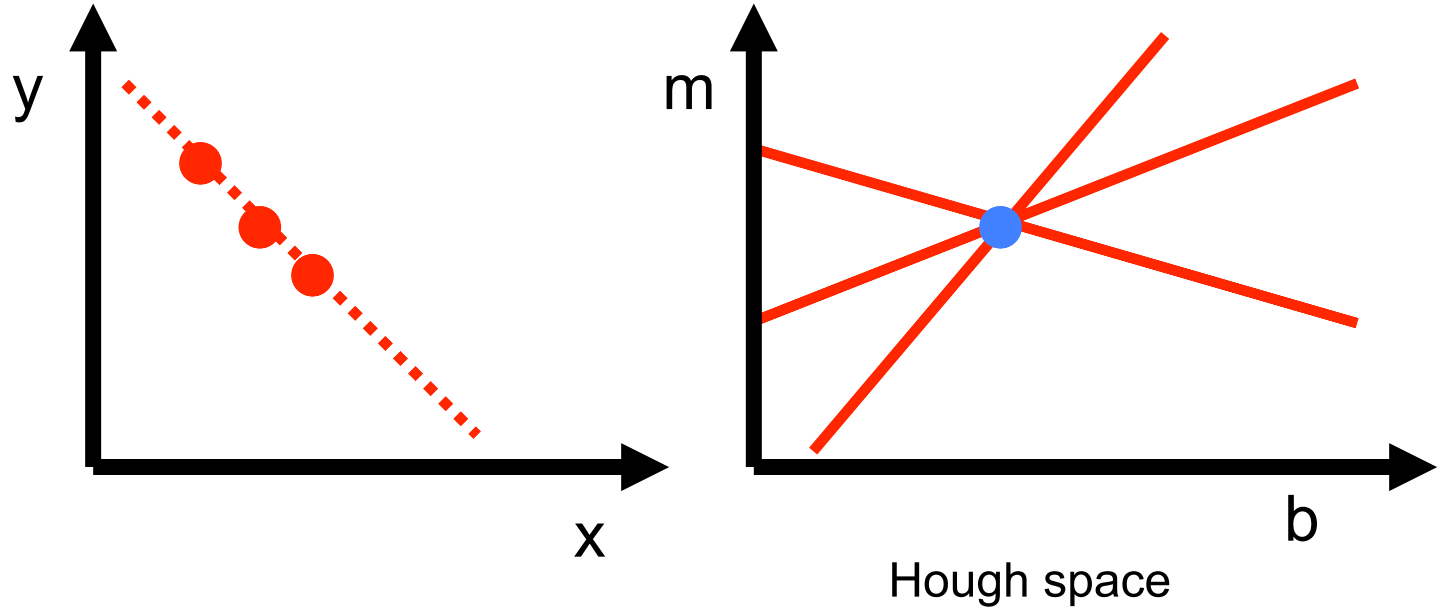
2. Each point votes for a set of parameters, incrementing those values in grid

3. Find maximum or local maxima in grid

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

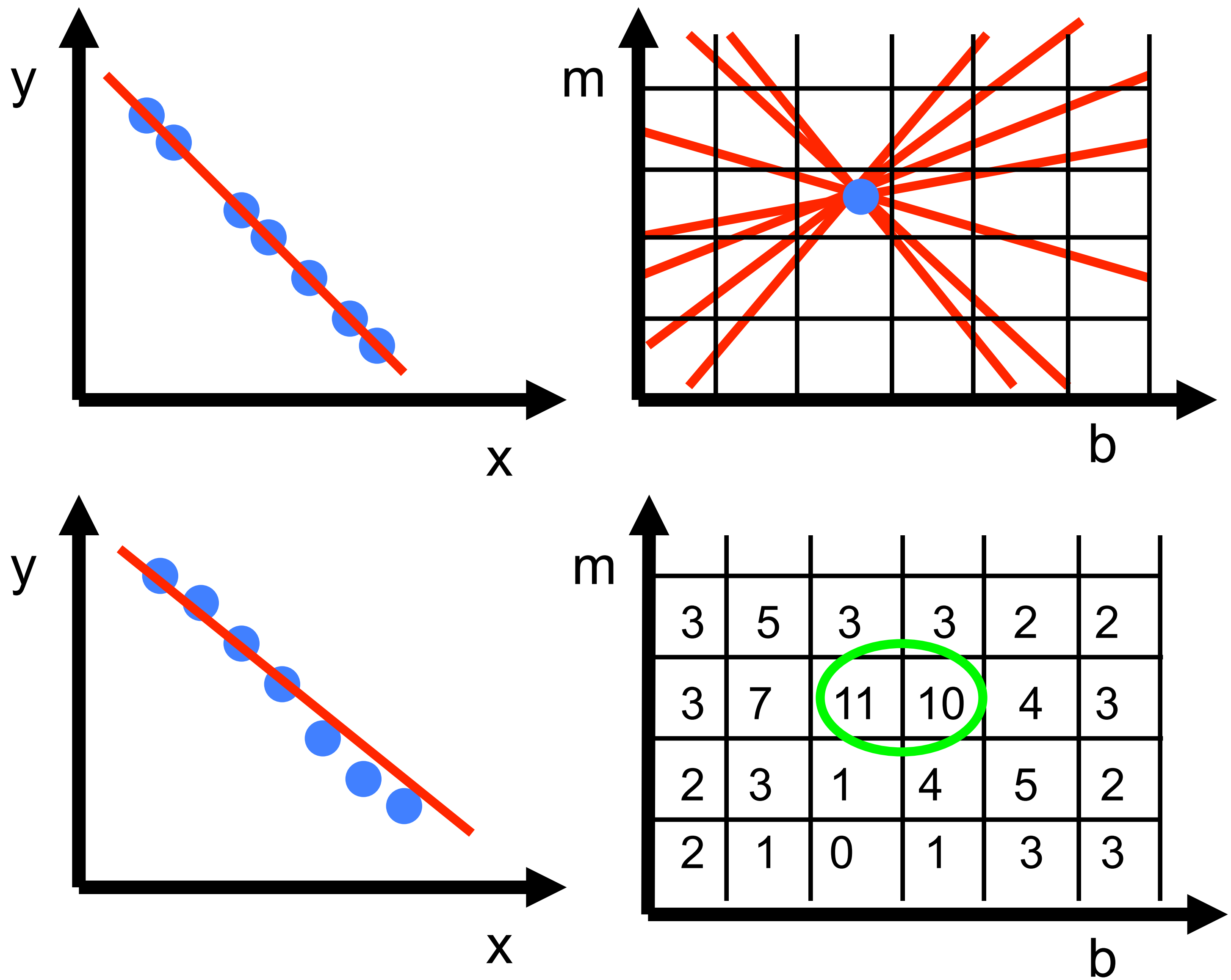
Given a set of points, find the curve or line that explains the data points best



$$y = m\,x + b$$

Slide from S. Savarese

Hough transform



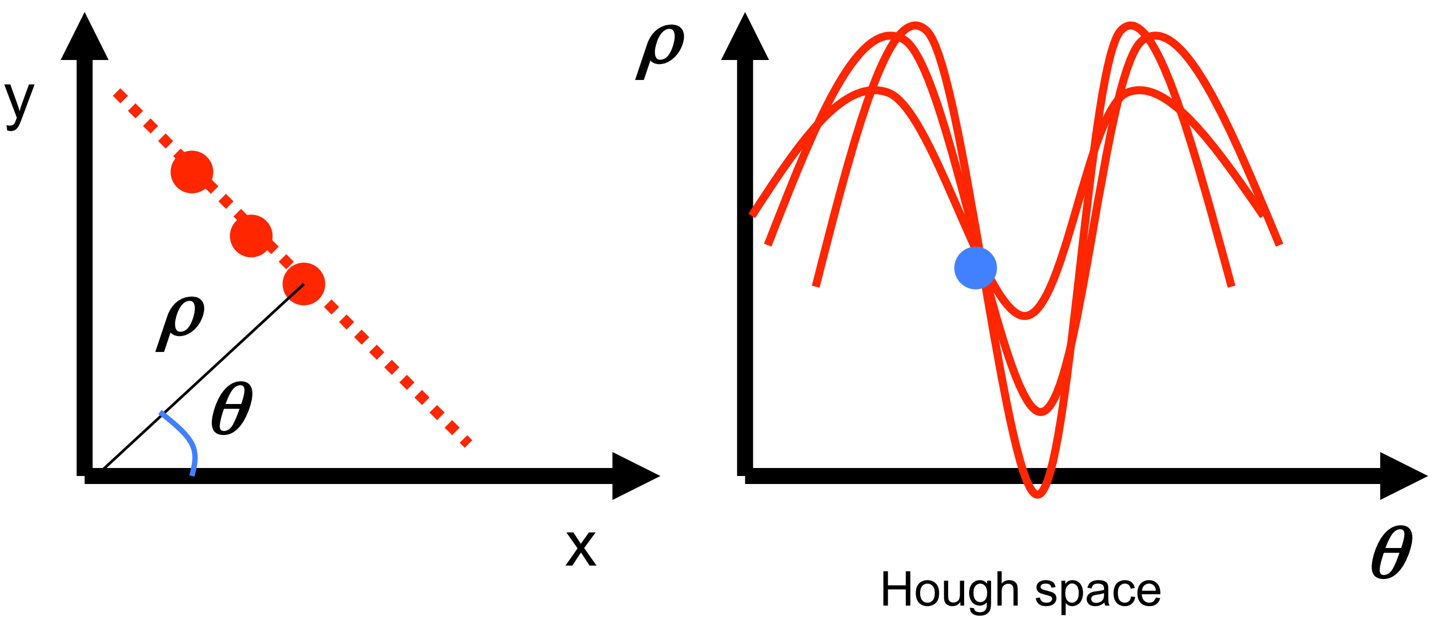
Slide from S. Savarese

Hough transform

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures,* Proc. Int. Conf. High Energy Accelerators and Instrumentation, 1959

Issue : parameter space [m,b] is unbounded…

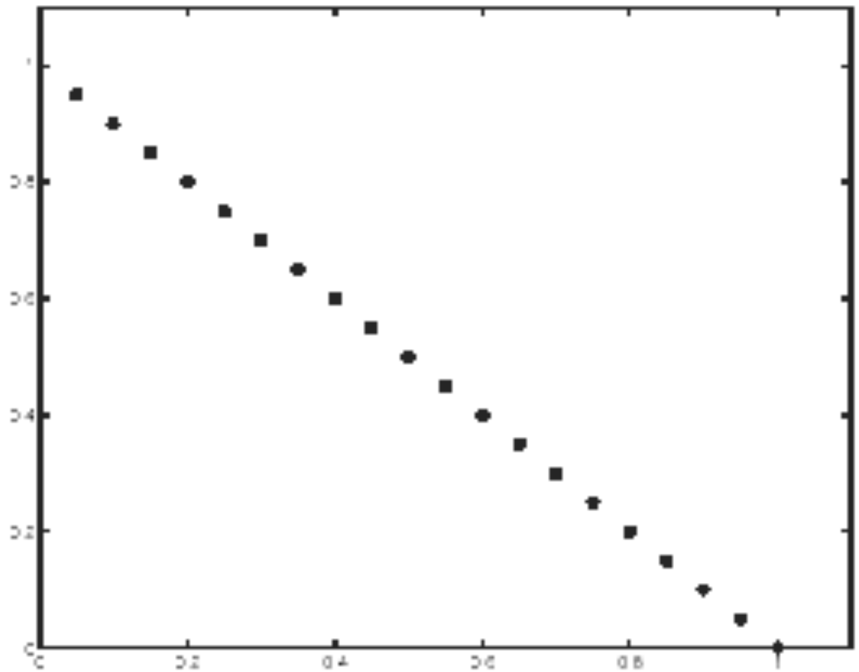
Use a polar representation for the parameter space



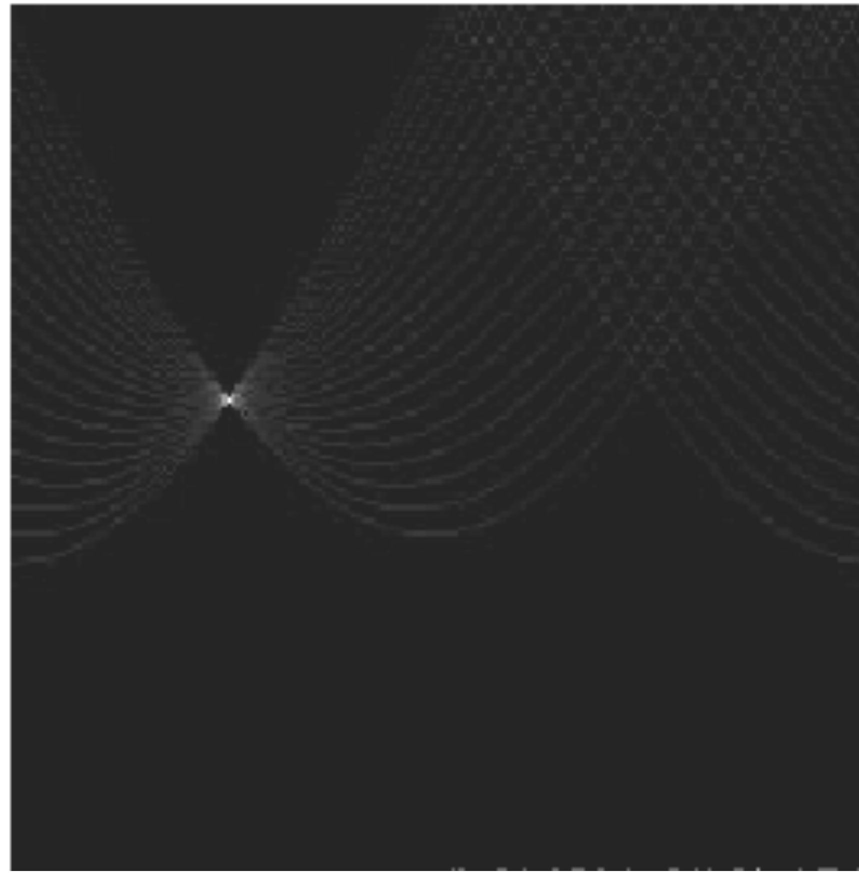
$$x\cos\theta + y\sin\theta = \rho$$

Slide from S. Savarese

Hough transform - experiments



features

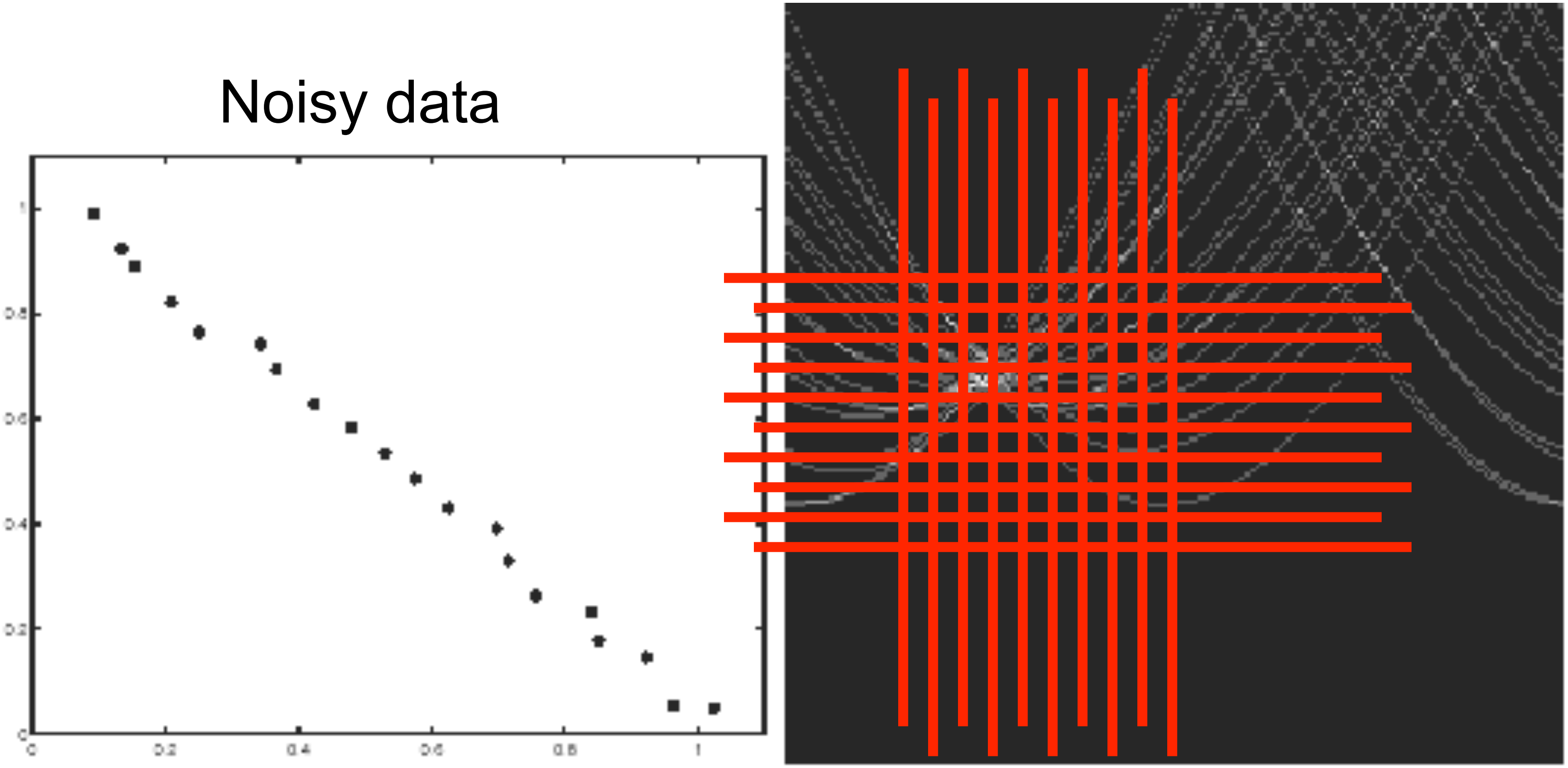


votes

Slide from S. Savarese

Hough transform - experiments

Noisy data

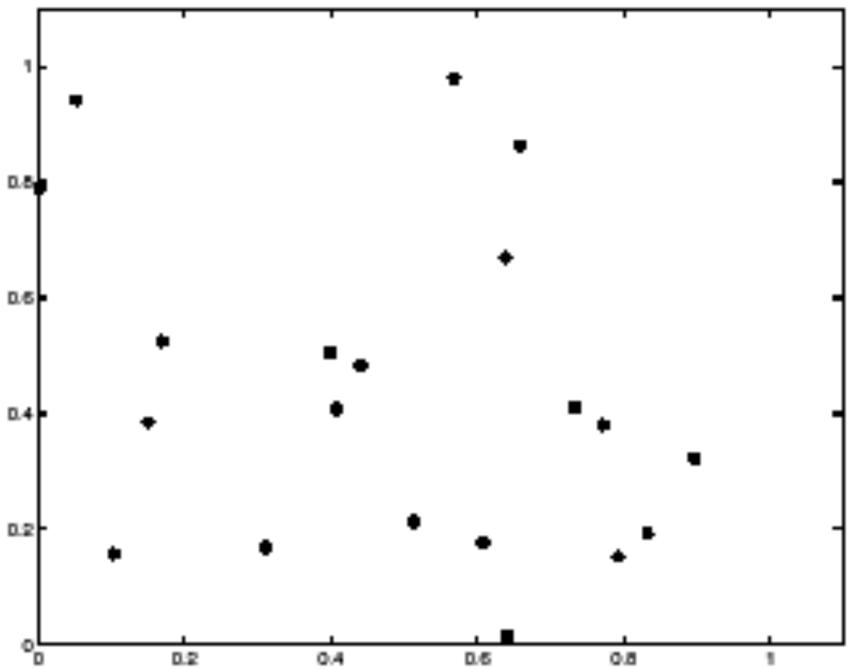


features votes

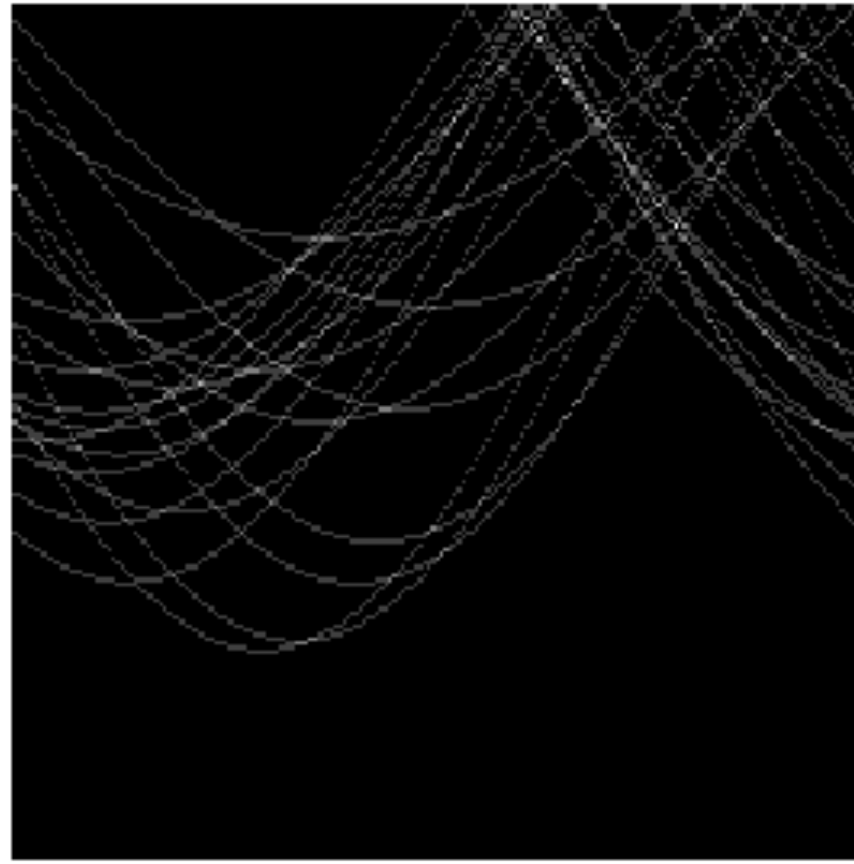
Need to adjust grid size or smooth

Slide from S. Savarese

Hough transform - experiments



features



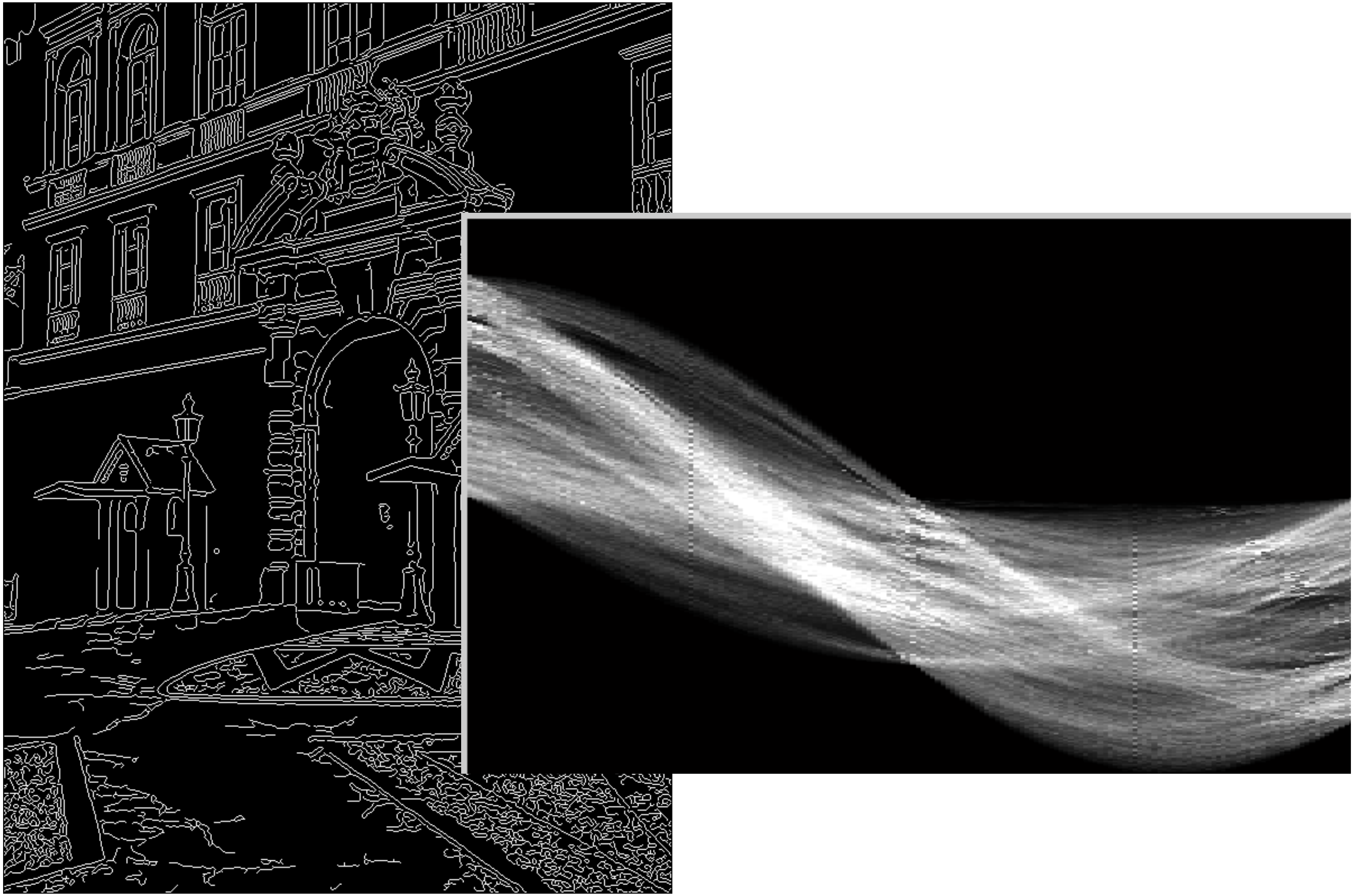
votes

Issue: spurious peaks due to uniform noise

Slide from S. Savarese

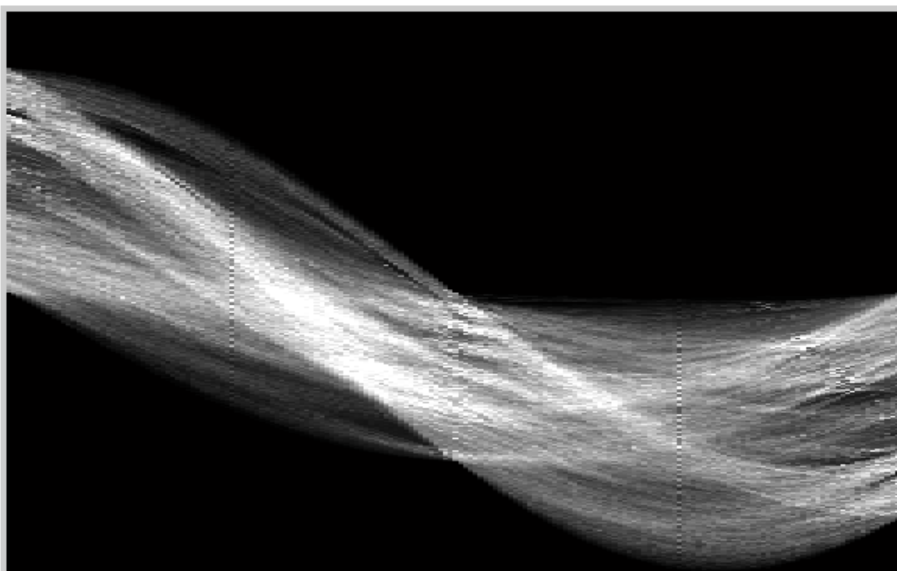
1. Image → Canny

2. Canny → Hough votes

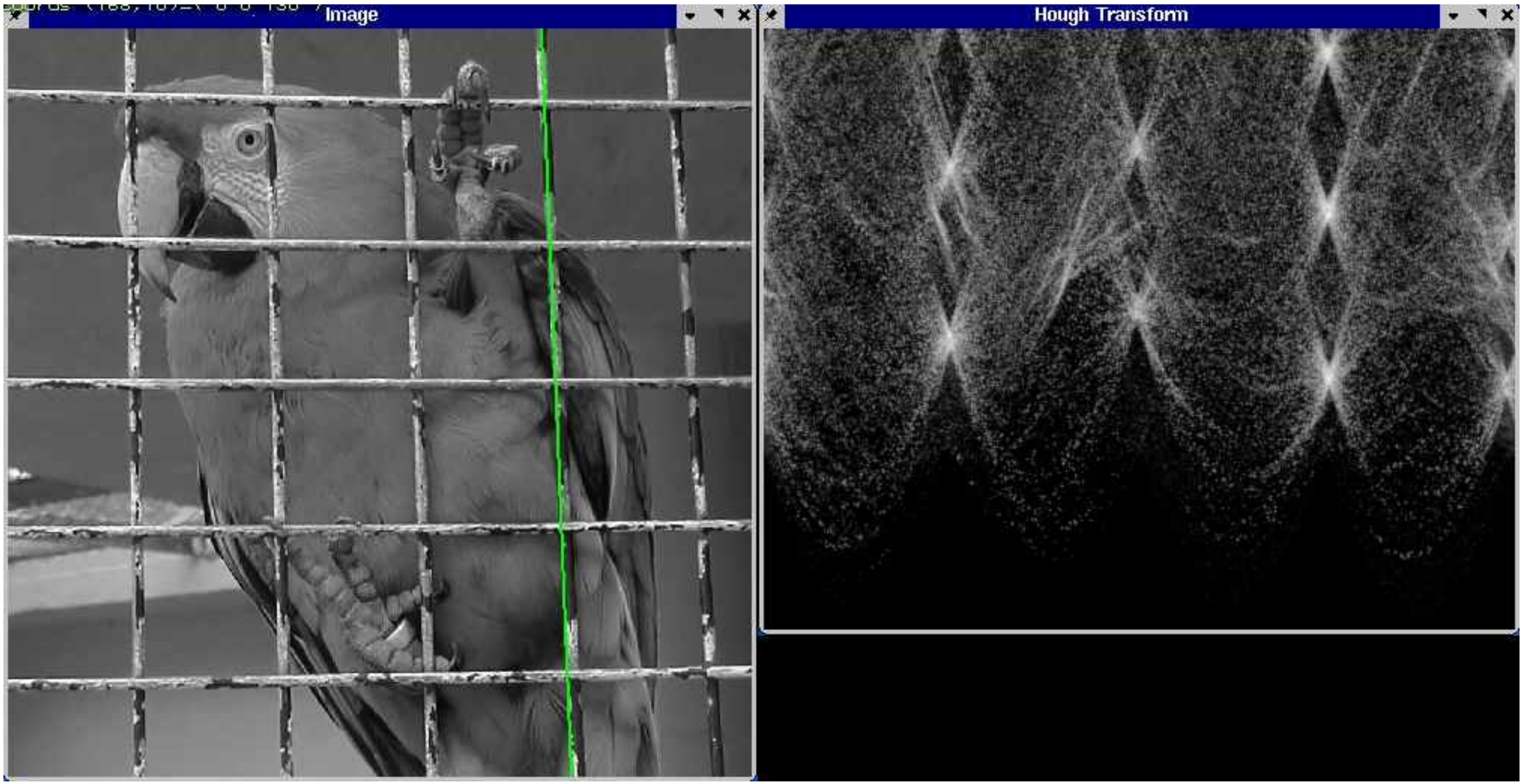


3. Hough votes → Edges

Find peaks and post-process



Hough transform example



http://ostatic.com/files/images/ss_hough.jpg

Finding lines using Hough transform

- Using m,b parameterization
- Using r, theta parameterization
  - Using oriented gradients
- Practical considerations
  - Bin size
  - Smoothing
  - Finding multiple lines
  - Finding line segments

Hough transform conclusions

Good

- Robust to outliers: each point votes separately
- Fairly efficient (much faster than trying all sets of parameters)
- Provides multiple good fits

Bad

- Some sensitivity to noise
- Bin size trades off between noise tolerance, precision, and speed/memory
  - Can be hard to find sweet spot
- Not suitable for more than a few parameters
  - grid size grows exponentially

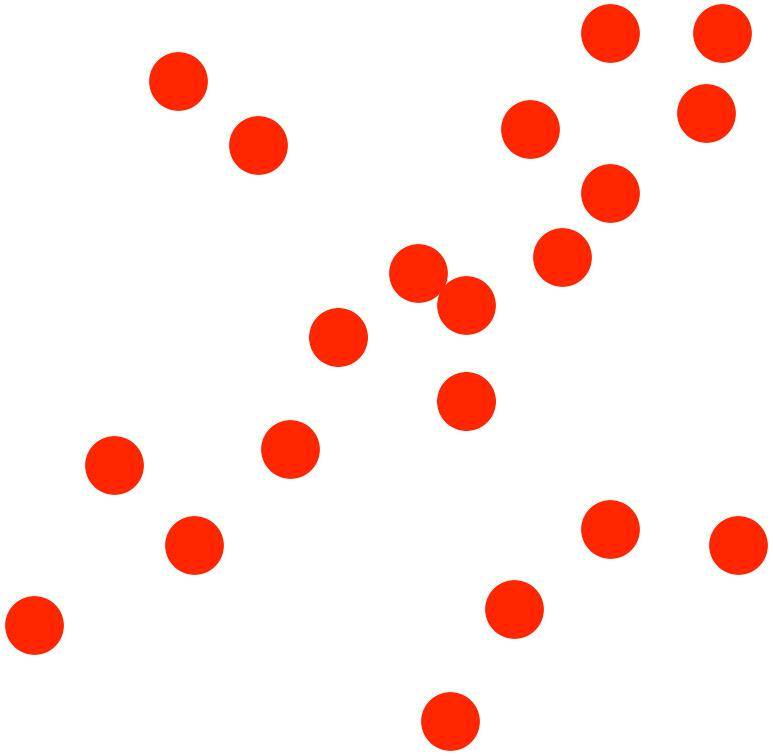
Common applications

- Line fitting (also circles, ellipses, etc.)
- Object instance recognition (parameters are affine transform)
- Object category recognition (parameters are position/scale)

RANSAC

(RANdom SAmple Consensus) :

Fischler & Bolles in '81.



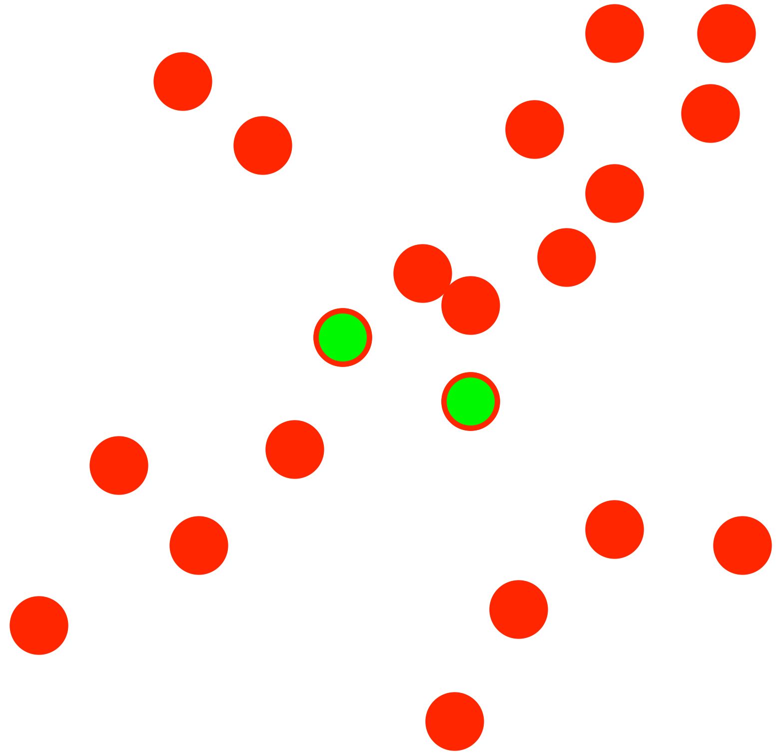
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



Algorithm:

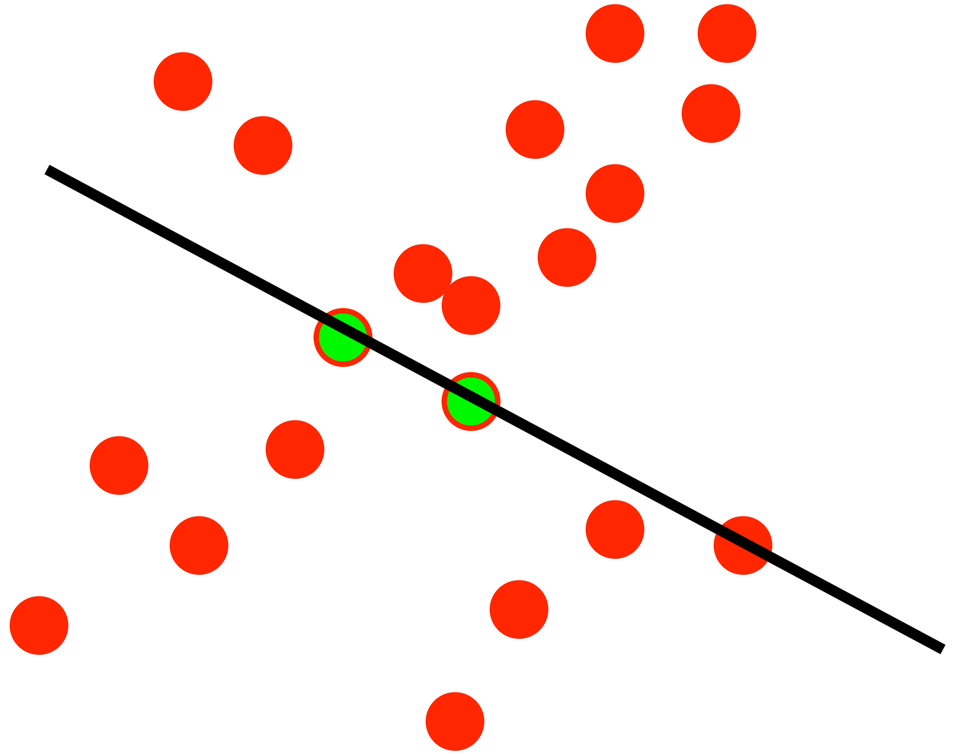
1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

Illustration by Savarese

RANSAC

Line fitting example



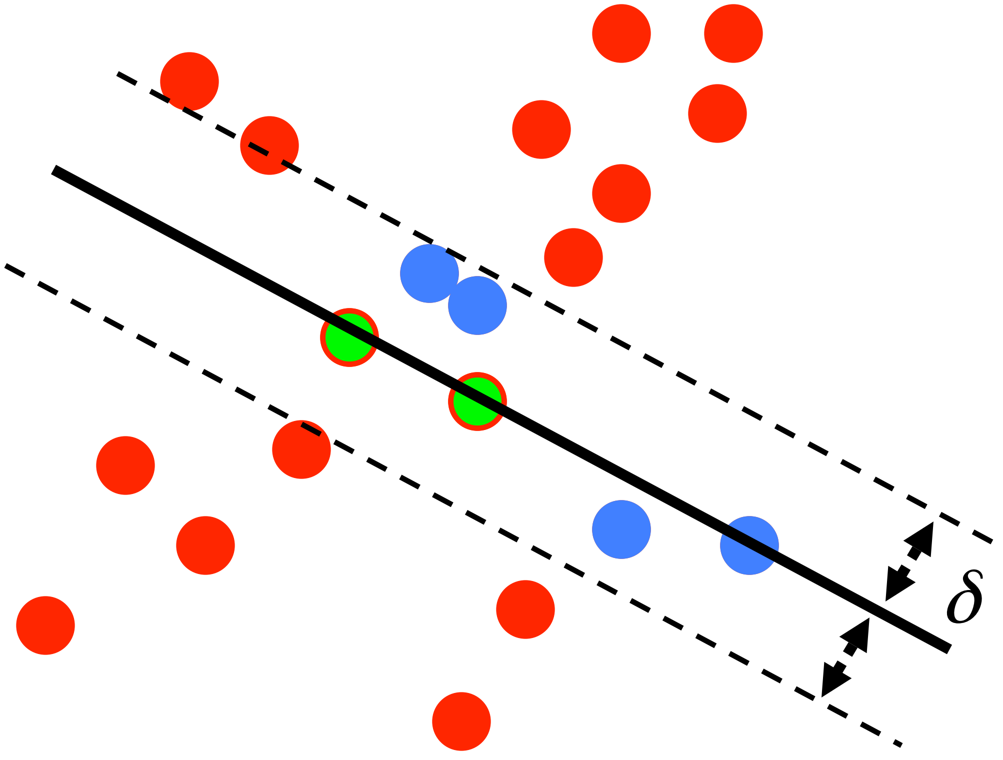
Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC

Line fitting example



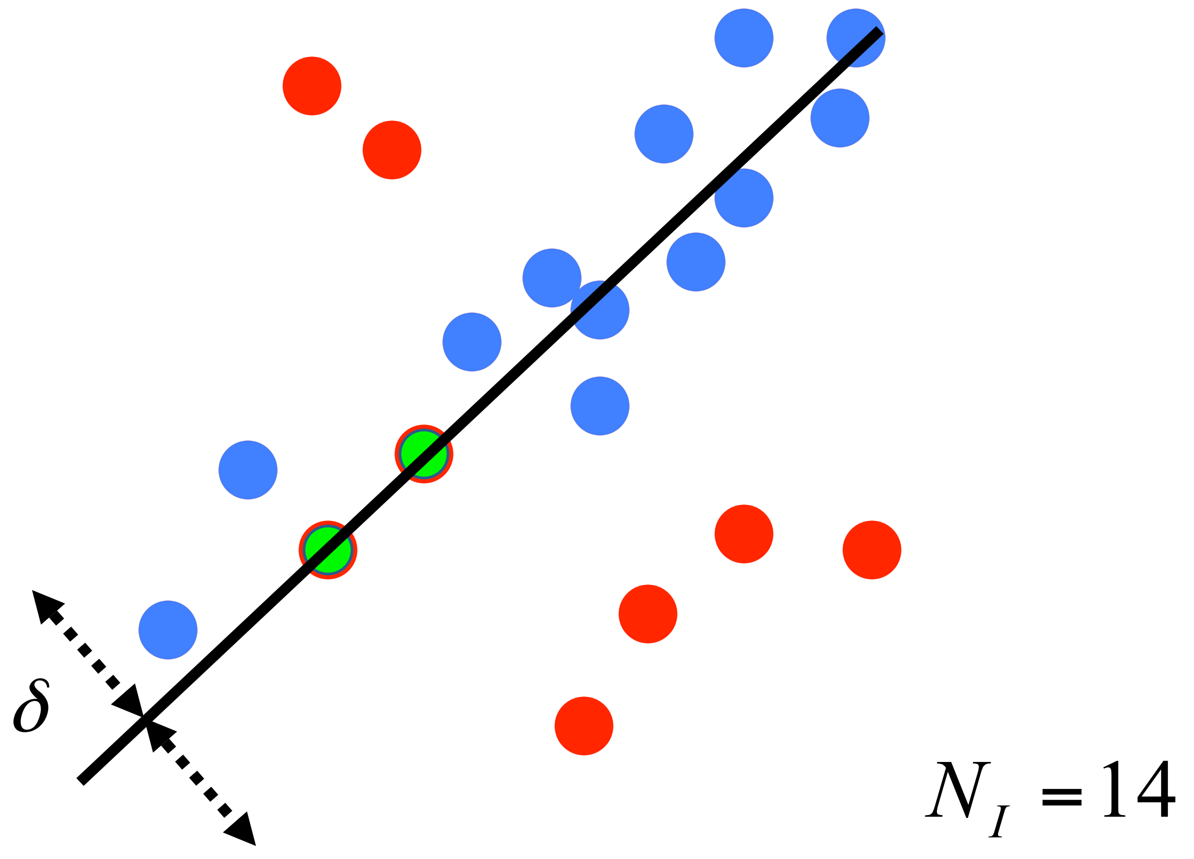
$$N_I = 6$$

Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

RANSAC



Algorithm:

1. **Sample** (randomly) the number of points required to fit the model (#=2)
2. **Solve** for model parameters using samples
3. **Score** by the fraction of inliers within a preset threshold of the model

Repeat 1-3 until the best model is found with high confidence

How to choose parameters?

- Number of samples *N*
  - Choose *N* so that, with probability *p*, at least one random sample is free from outliers (e.g. *p*=0.99) (outlier ratio: *e*)
- Number of sampled points *s*
  - Minimum number needed to fit the model
- Distance threshold δ
  - Choose δ so that a good point with noise is likely (e.g., prob=0.95) within threshold
  - Zero-mean Gaussian noise with std. dev. σ: $t^2=3.84\sigma^2$

$$N = \log(1-p)/\log\left(1-(1-e)^s\right)$$

	proportion of outliers *e*						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

modified from M. Pollefeys

RANSAC conclusions

Good

- Robust to outliers
- Applicable for larger number of objective function parameters than Hough transform
- Optimization parameters are easier to choose than Hough transform

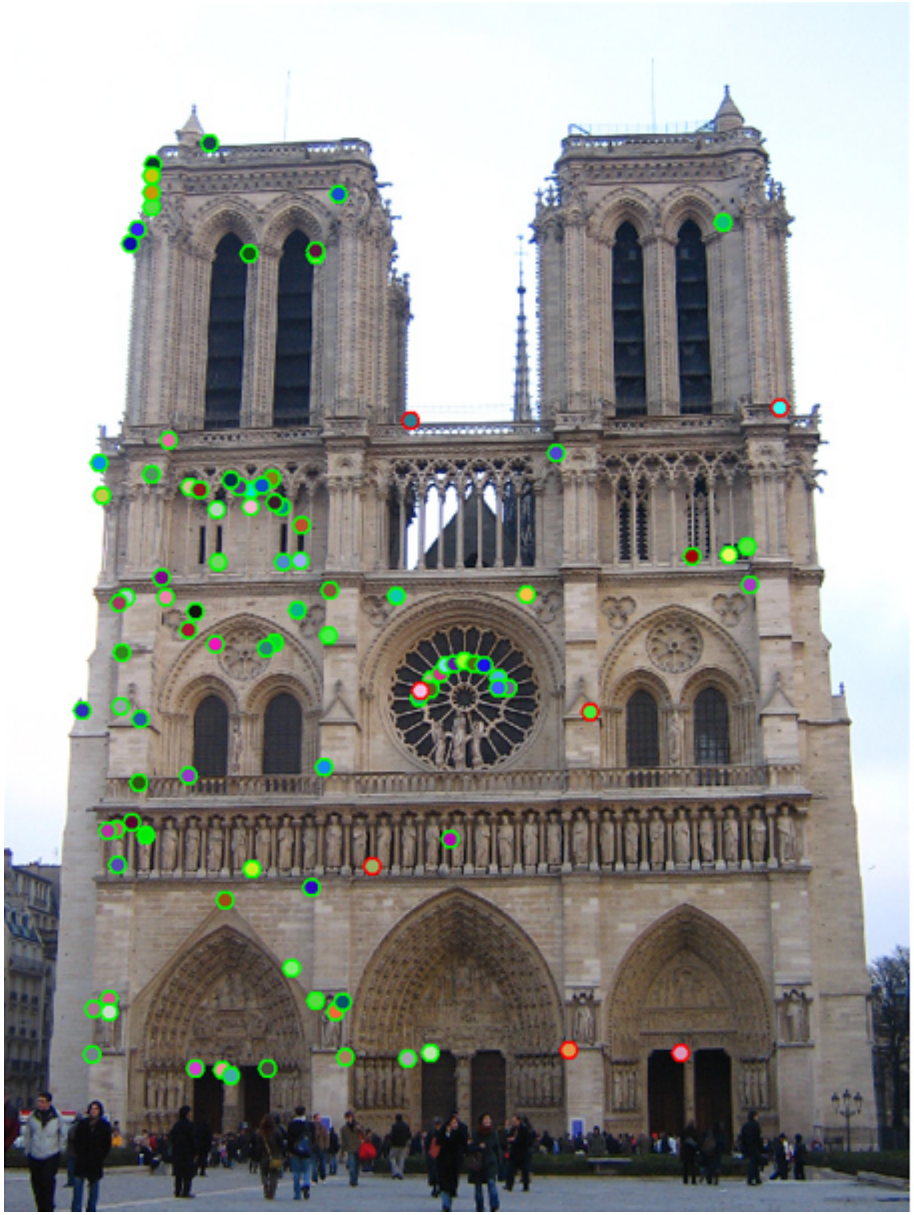
Bad

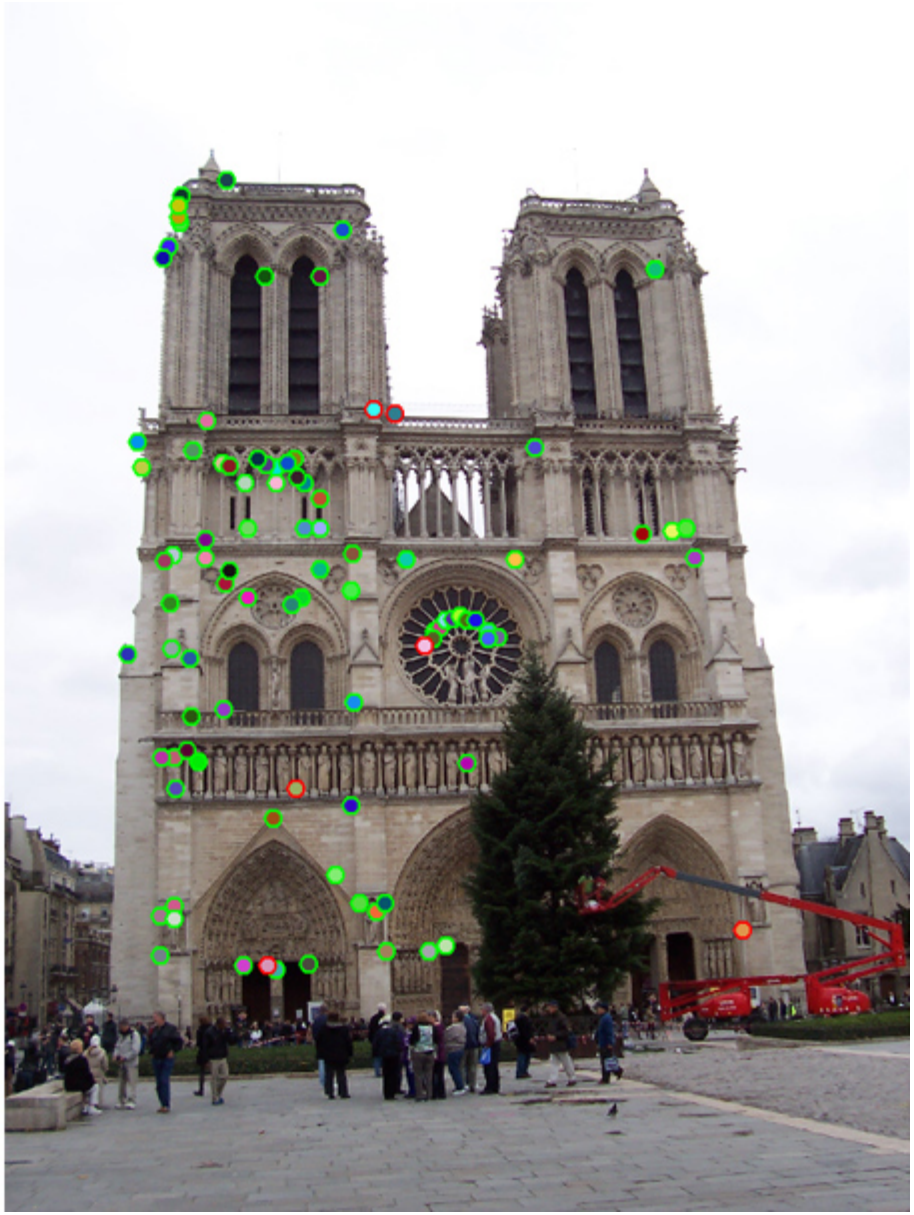
- Computational time grows quickly with fraction of outliers and number of parameters
- Not good for getting multiple fits

Common applications

- Computing a homography (e.g., image stitching)
- Estimating fundamental matrix (relating two views)

How do we fit the best alignment?





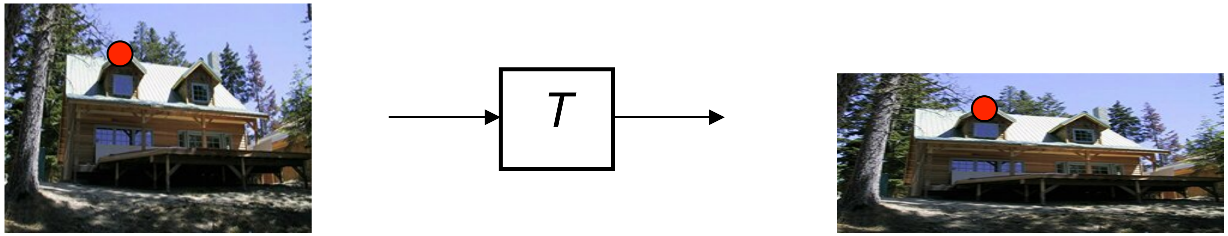
Alignment

- Alignment: find parameters of model that maps one set of points to another

- Typically want to solve for a global transformation that accounts for *most* true correspondences

- Difficulties
  - Noise (typically 1-3 pixels)
  - Outliers (often 50%)
  - Many-to-one matches or multiple objects

Parametric (global) warping



p = (x,y) **p'** = (x',y')

Transformation T is a coordinate-changing machine:

p' = *T*(p)

What does it mean that *T* is global?

– Is the same for any point p
– can be described by just a few numbers (parameters)

For linear transformations, we can represent T as a matrix

p' = **T**p

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{T} \begin{bmatrix} x \\ y \end{bmatrix}$$

Common transformations

original

Transformed

translation

rotation

aspect

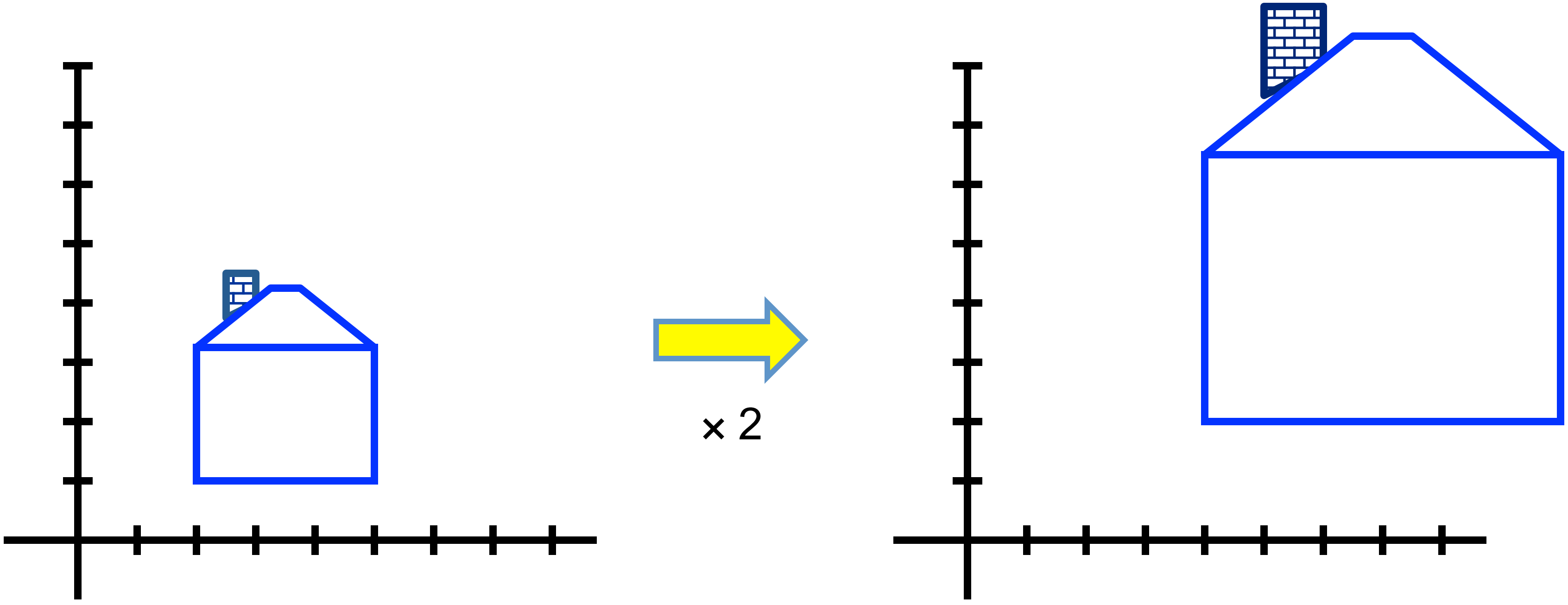
affine

perspective

Slide credit (next few slides):
A. Efros and/or S. Seitz

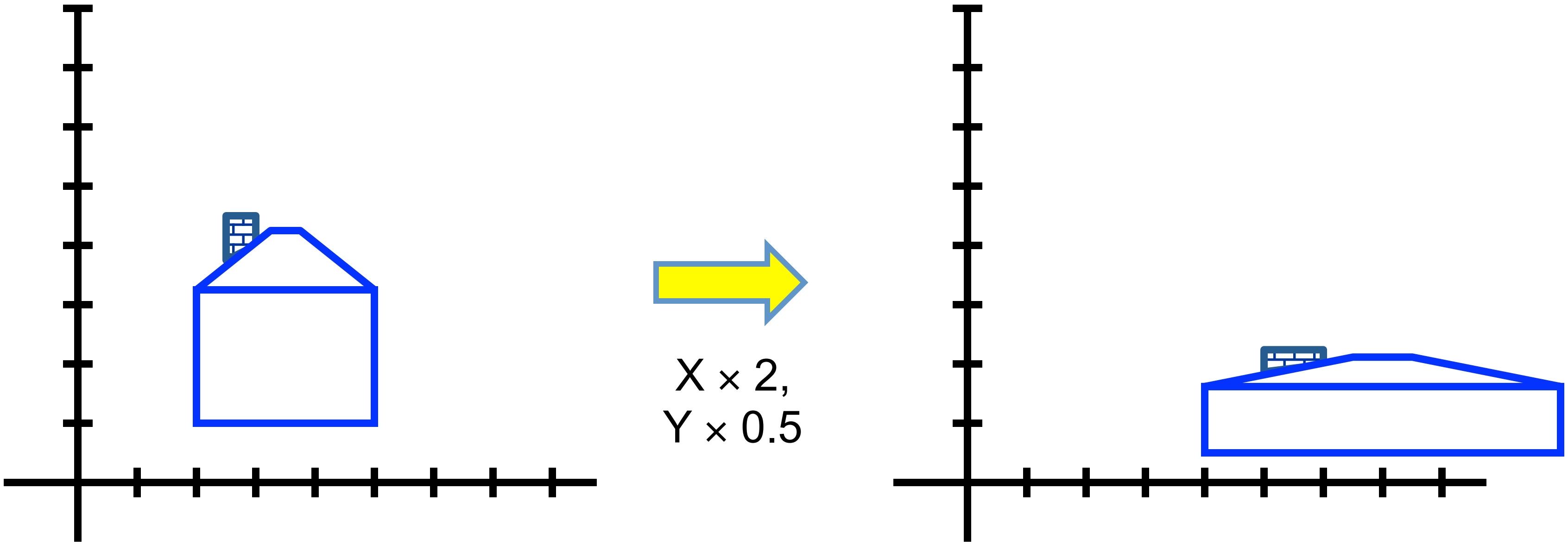
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

- Scaling operation:

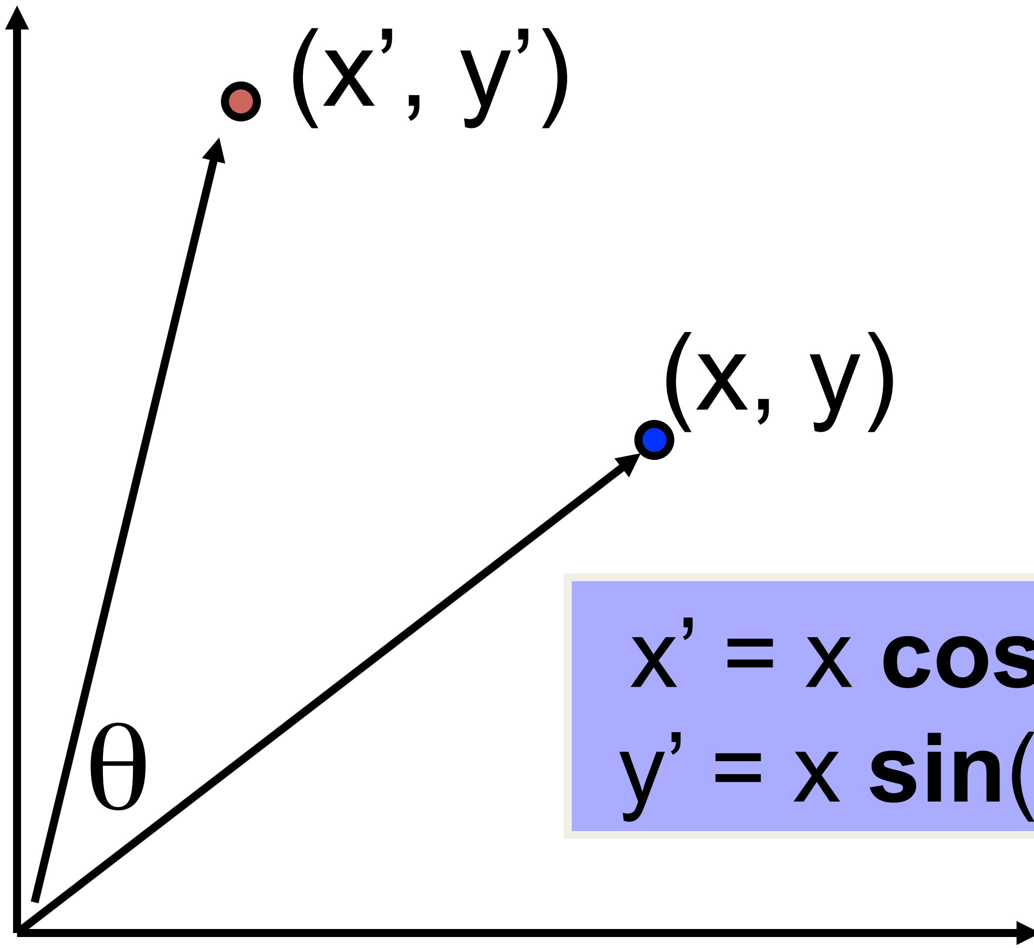
$$x' = ax$$

$$y' = by$$

- Or, in matrix form:

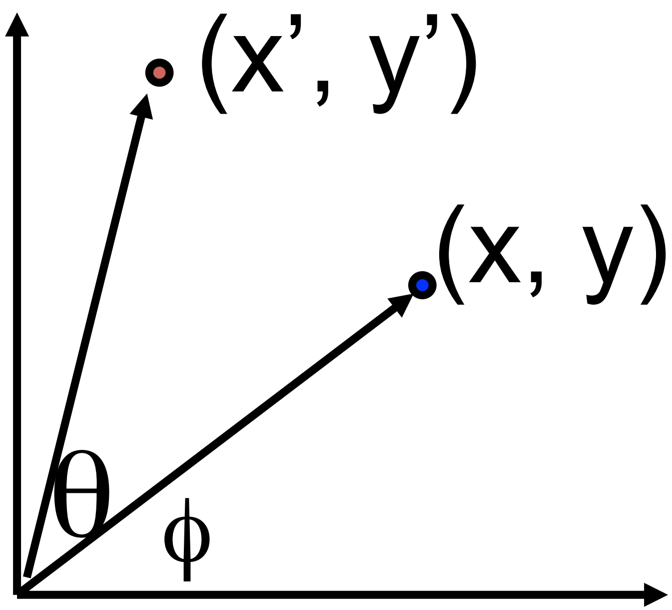
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\textit{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

2-D Rotation



$$x' = x\,\mathbf{cos}(\theta) - y\,\mathbf{sin}(\theta)$$
$$y' = x\,\mathbf{sin}(\theta) + y\,\mathbf{cos}(\theta)$$

2-D Rotation



Polar coordinates…

$x = r\cos(\phi)$

$y = r\sin(\phi)$

$x' = r\cos(\phi + \theta)$

$y' = r\sin(\phi + \theta)$

Trig Identity…

$x' = r\cos(\phi)\cos(\theta) - r\sin(\phi)\sin(\theta)$

$y' = r\sin(\phi)\cos(\theta) + r\cos(\phi)\sin(\theta)$

Substitute…

$x' = x\,\mathbf{cos}(\theta) - y\,\mathbf{sin}(\theta)$

$y' = x\,\mathbf{sin}(\theta) + y\,\mathbf{cos}(\theta)$

2-D Rotation

This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Even though sin(θ) and cos(θ) are nonlinear functions of θ,

- ***x' is a linear combination of x and y***
- ***y' is a linear combination of x and y***

What is the inverse transformation?

- Rotation by –θ
- For rotation matrices $\mathbf{R}^{-1} = \mathbf{R}^T$

Basic 2D transformations

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \alpha_x \\ \alpha_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Shear

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine

Affine is any combination of translation, scale, rotation, shear

Affine Transformations

Affine transformations are combinations of

- Linear transformations, and
- Translations

Properties of affine transformations:

- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

or

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Projective Transformations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

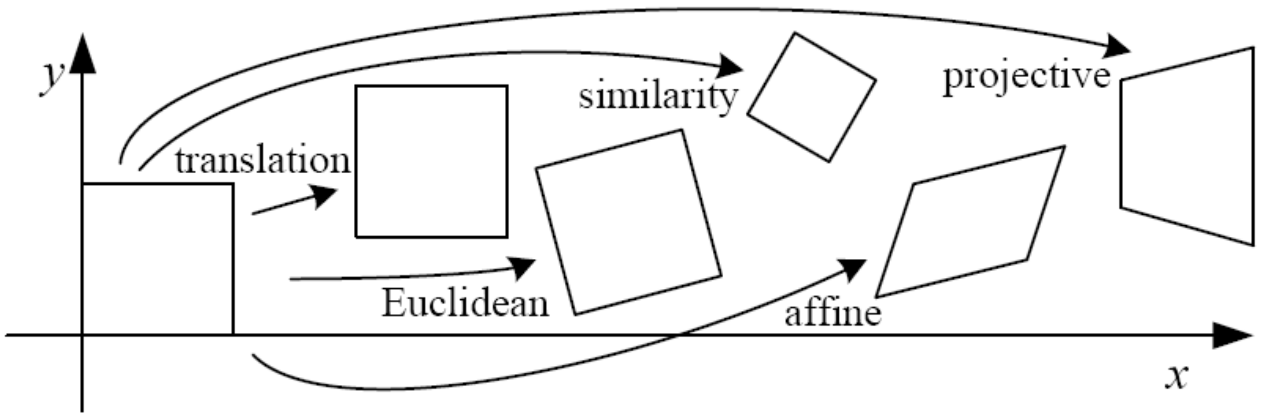
Projective transformations are combos of

- Affine transformations, and
- Projective warps

Properties of projective transformations:

- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)

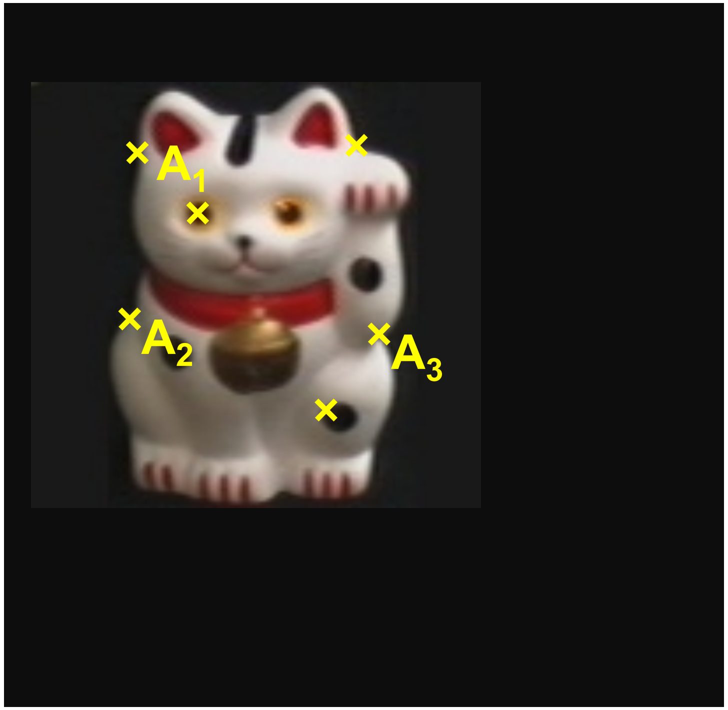
2D image transformations (reference table)



Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

Szeliski 2.1

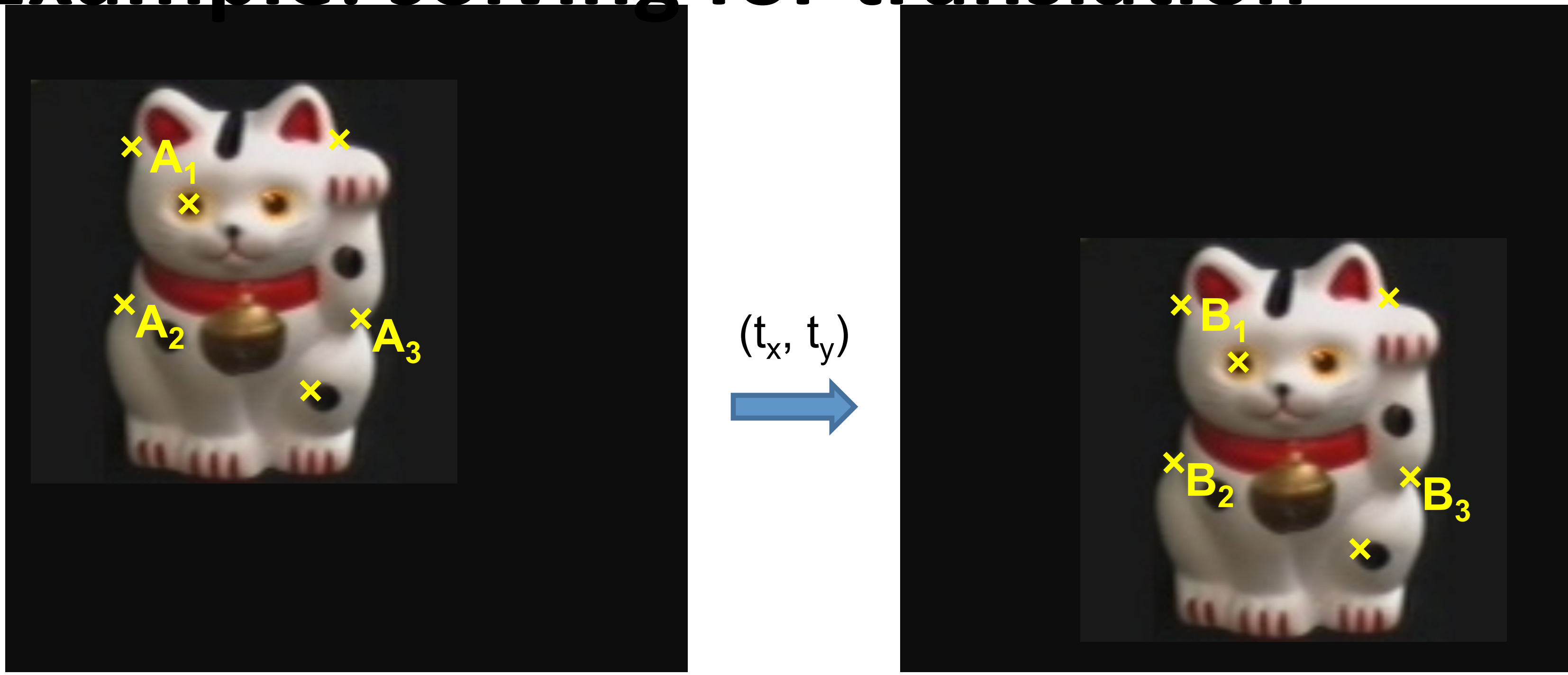
Example: solving for translation



Given matched points in {A} and {B}, estimate the translation of the object

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



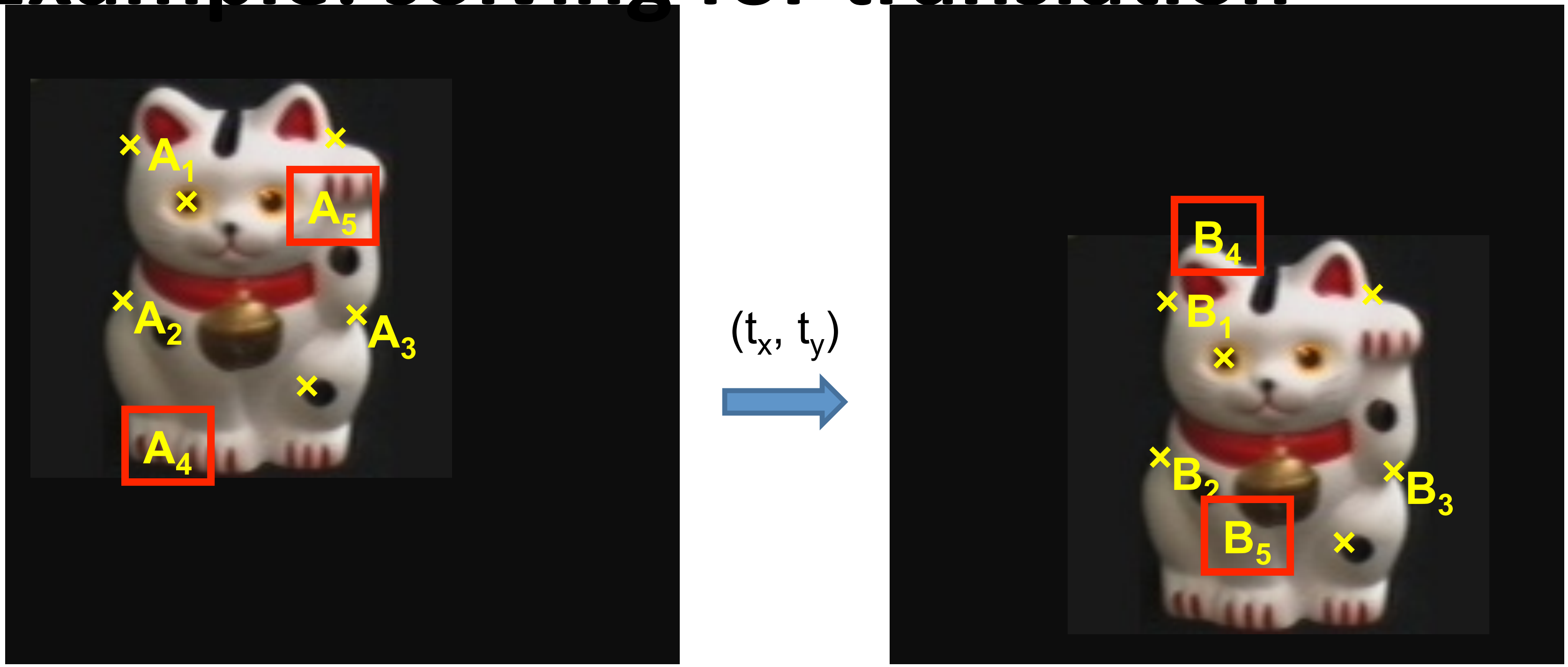
Least squares solution

1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form Ax=b
   b) Solve using pseudo-inverse or eigenvalue decomposition

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_1^B - x_1^A \\ y_1^B - y_1^A \\ \vdots \\ x_n^B - x_n^A \\ y_n^B - y_n^A \end{bmatrix}$$

Example: solving for translation



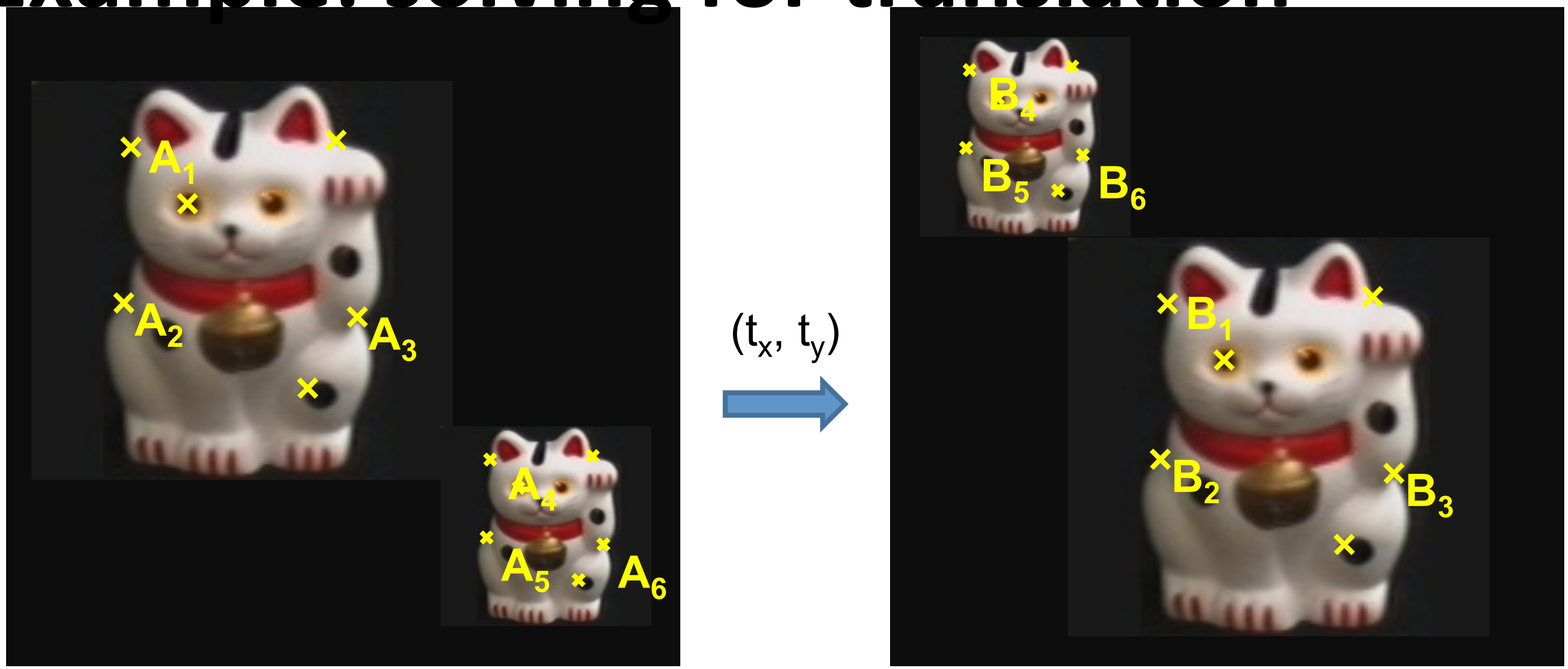
Problem: outliers

RANSAC solution

1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation



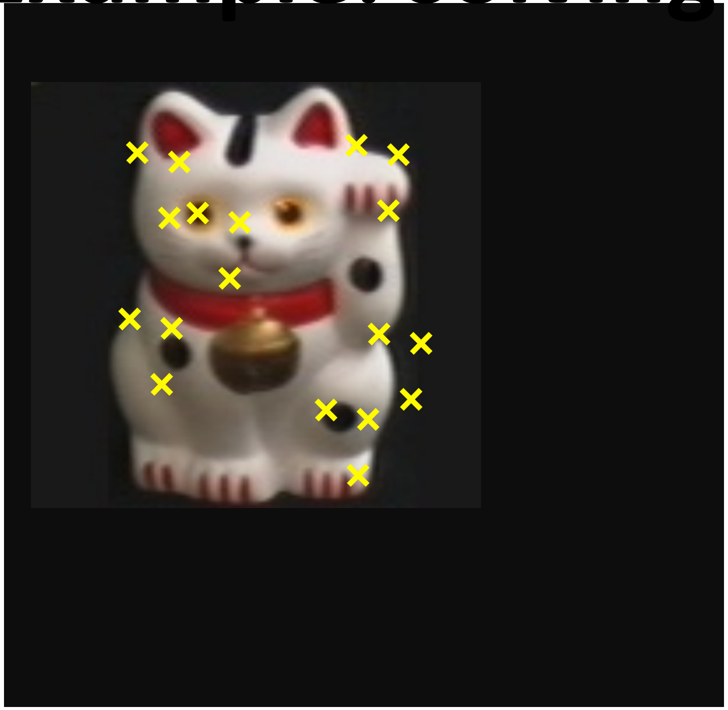
Problem: outliers, multiple objects, and/or many-to-one matches

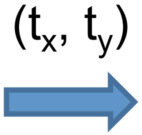
Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Example: solving for translation





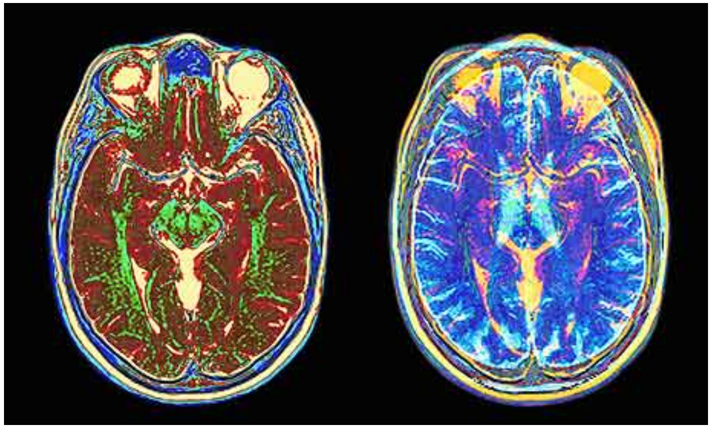
Problem: no initial guesses for correspondence

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

What if you want to align but have no prior matched pairs?

- Hough transform and RANSAC not applicable

- Important applications



Medical imaging: match brain scans or contours

Robotics: match point clouds

Iterative Closest Points (ICP) Algorithm

Goal: estimate transform between two dense sets of points

1. **Initialize** transformation (e.g., compute difference in means and scale)
2. **Assign** each point in {Set 1} to its nearest neighbor in {Set 2}
3. **Estimate** transformation parameters
   - e.g., least squares or robust least squares
4. **Transform** the points in {Set 1} using estimated parameters
5. **Repeat** steps 2-4 until change is very small

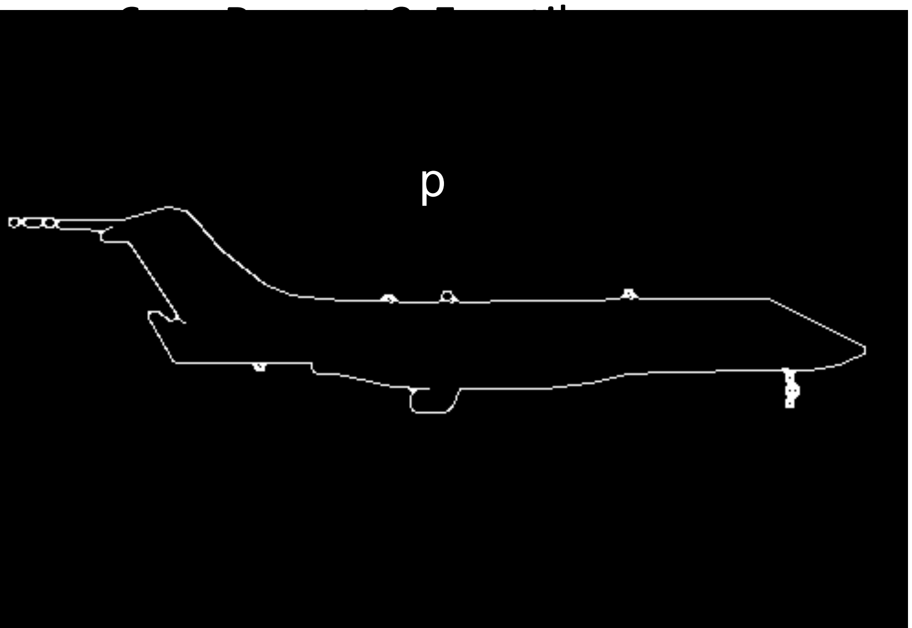
Example: aligning boundaries

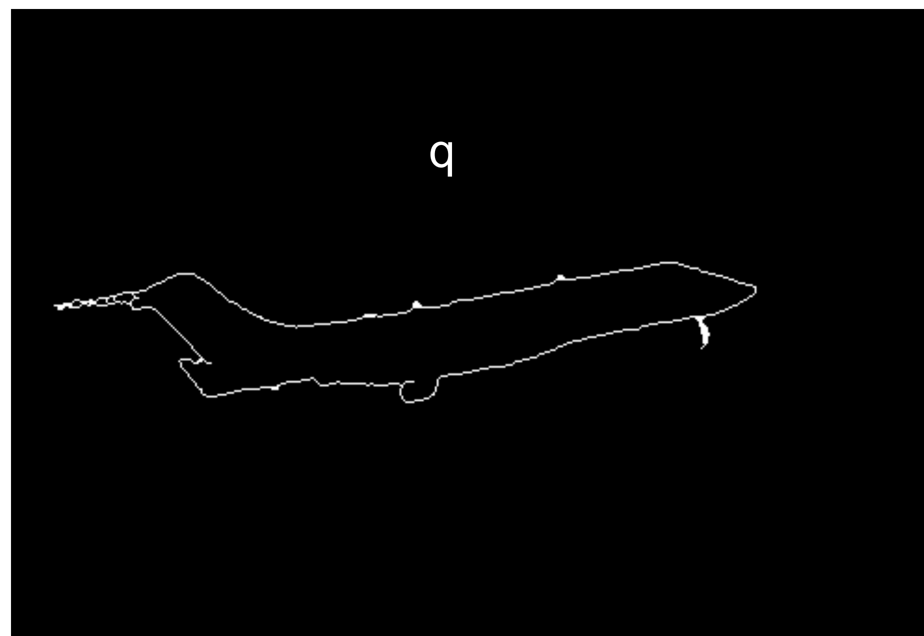
1. Extract edge pixels $p1..pn$ and $q1..qm$
2. Compute initial transformation (e.g., compute translation and scaling by center of mass, variance within each image)
3. Get nearest neighbors: for each point pi find corresponding $\mathrm{match(i)}=\mathrm{argmin}_j\ dist(pi,qj)$
4. Compute transformation ***T*** based on matches
5. Warp points ***p*** according to ***T***
6. Repeat 3-5 until convergence

Example: aligning boundaries

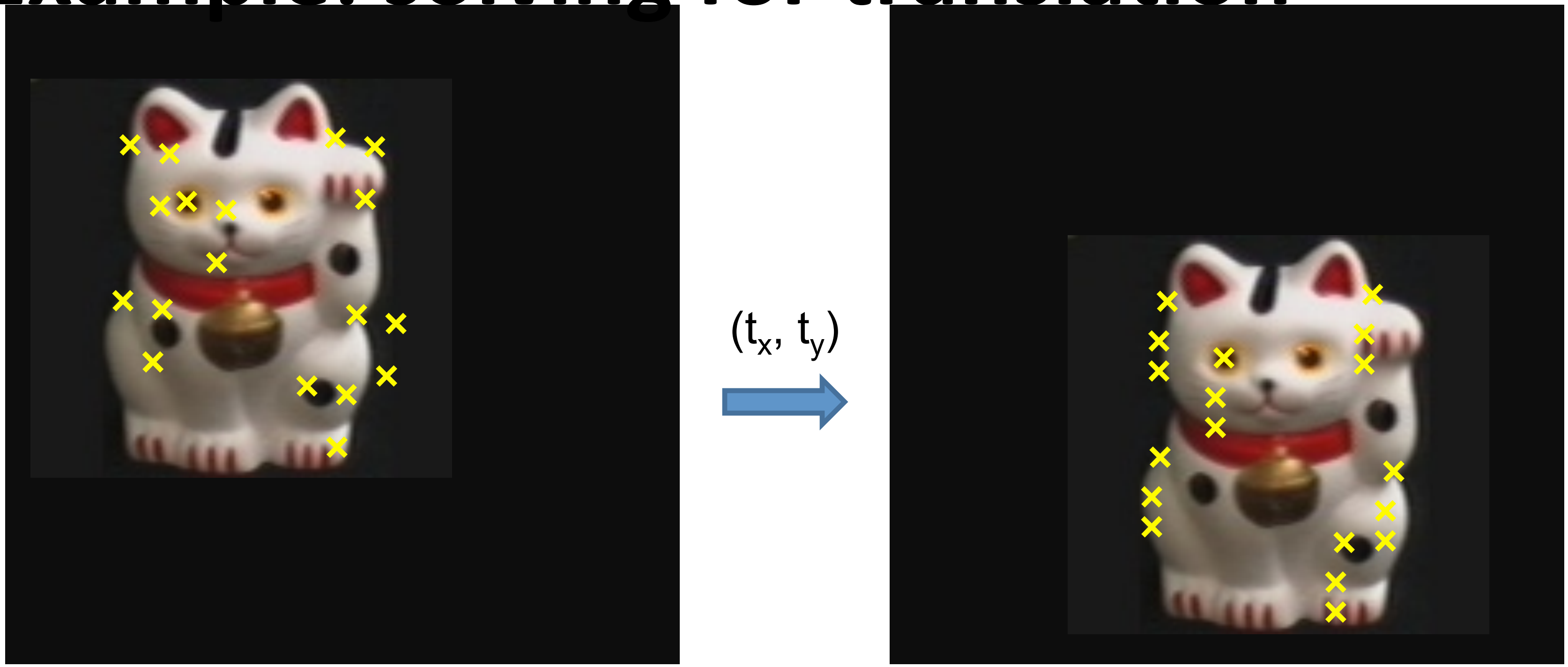
1. Extract edge pixels $p_1..p_n$ and $q_1..q_m$
2. Compute initial transformation (e.g., compute translation and scaling by center of mass, variance within each image)
3. Get nearest neighbors: for each point p_i find corresponding

   $\text{match}(i) = \text{argmin}_j \, dist(p_i, q_j)$
4. Compute transformation ***T*** based on matches
5. Warp points ***p*** according to ***T***





Example: solving for translation



Problem: no initial guesses for correspondence

ICP solution

1. Find nearest neighbors for each point
2. Compute transform using matches
3. Move points using transform
4. Repeat steps 1-3 until convergence

$$\begin{bmatrix} x_i^B \\ y_i^B \end{bmatrix} = \begin{bmatrix} x_i^A \\ y_i^A \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Algorithm Summary

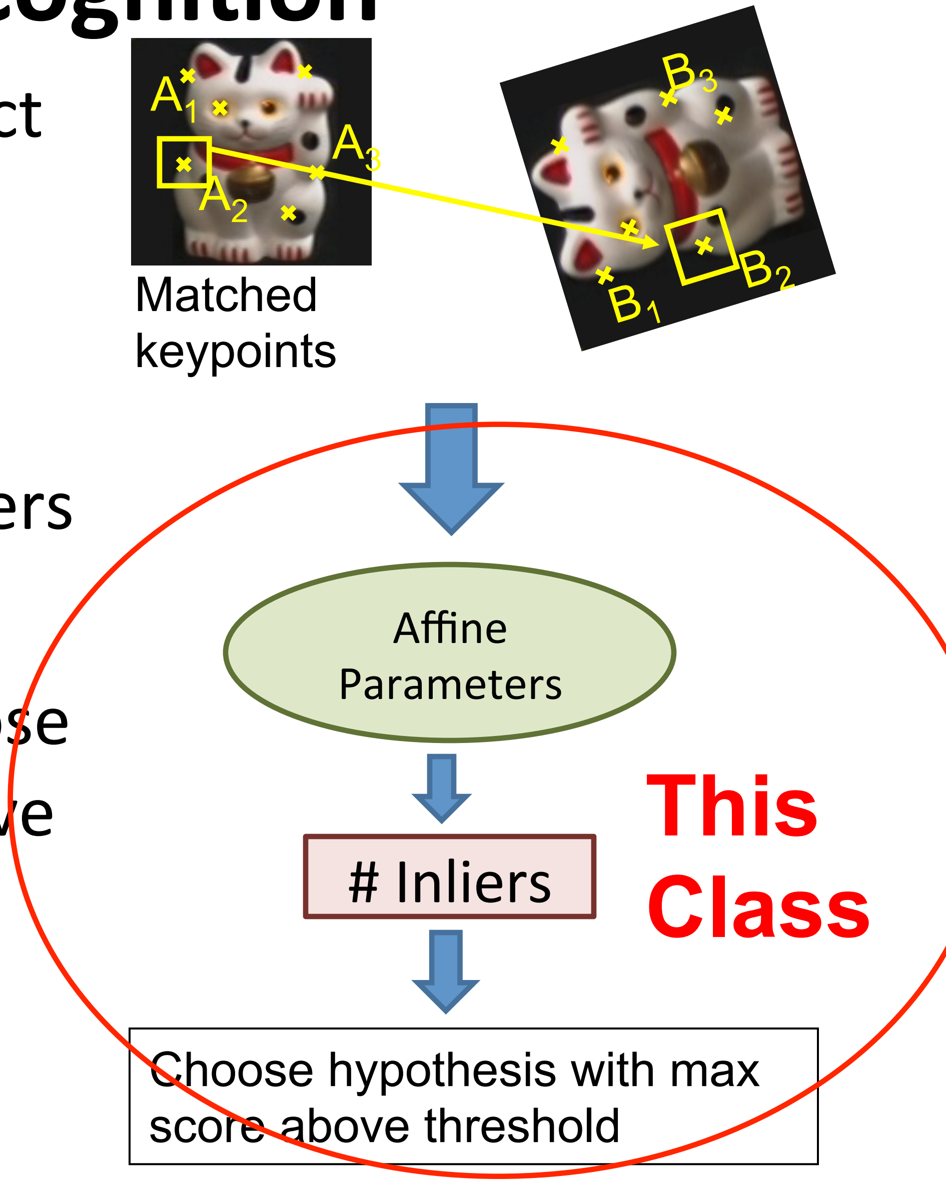
- Least Squares Fit
  - closed form solution
  - robust to noise
  - not robust to outliers
- Robust Least Squares
  - improves robustness to noise
  - requires iterative optimization
- Hough transform
  - robust to noise and outliers
  - can fit multiple models
  - only works for a few parameters (1-4 typically)
- RANSAC
  - robust to noise and outliers
  - works with a moderate number of parameters (e.g, 1-8)
- Iterative Closest Point (ICP)
  - For local alignment only: does not require initial correspondences

Object Instance Recognition

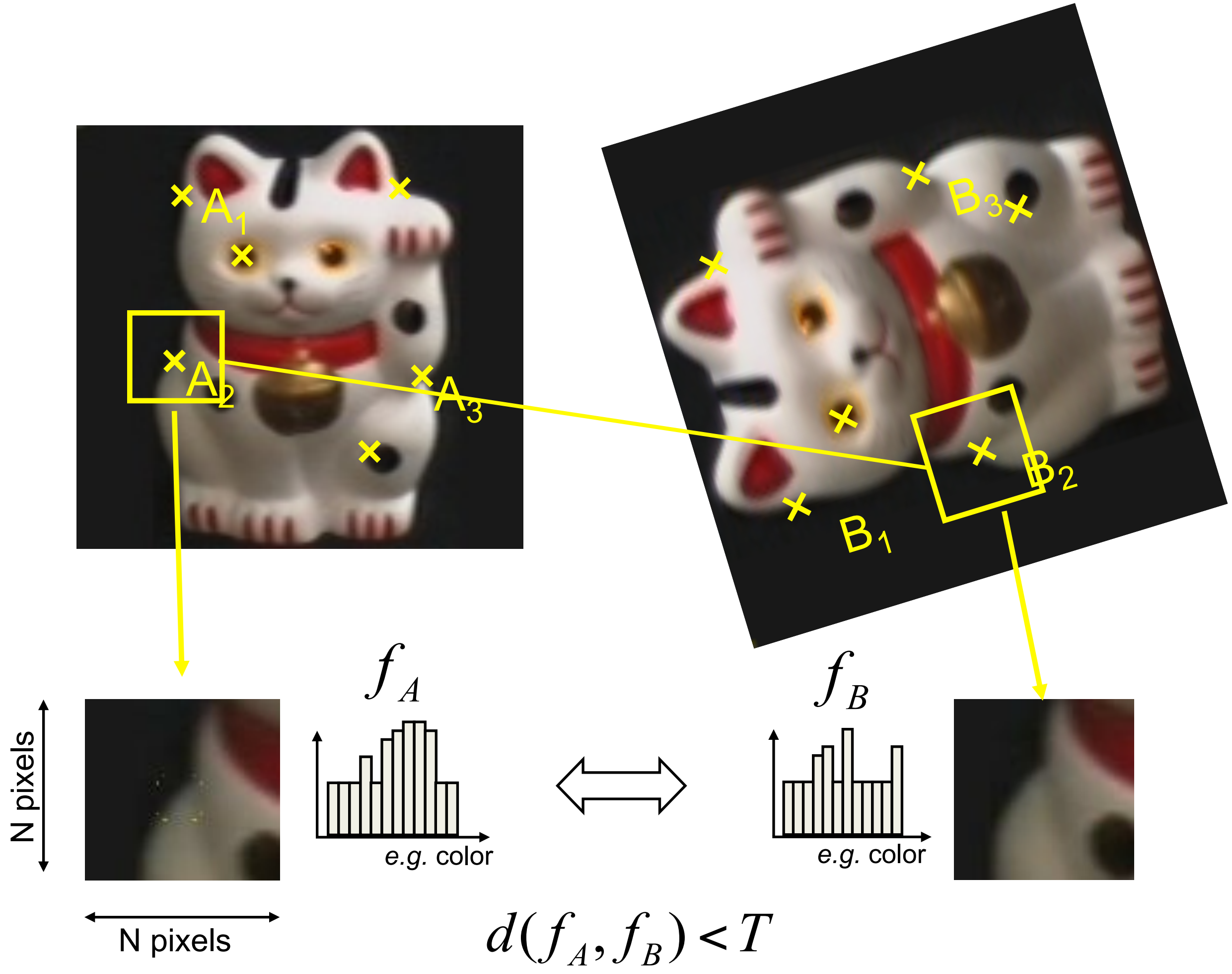
1. Match keypoints to object model

2. Solve for affine transformation parameters

3. Score by inliers and choose solutions with score above threshold



Overview of Keypoint Matching

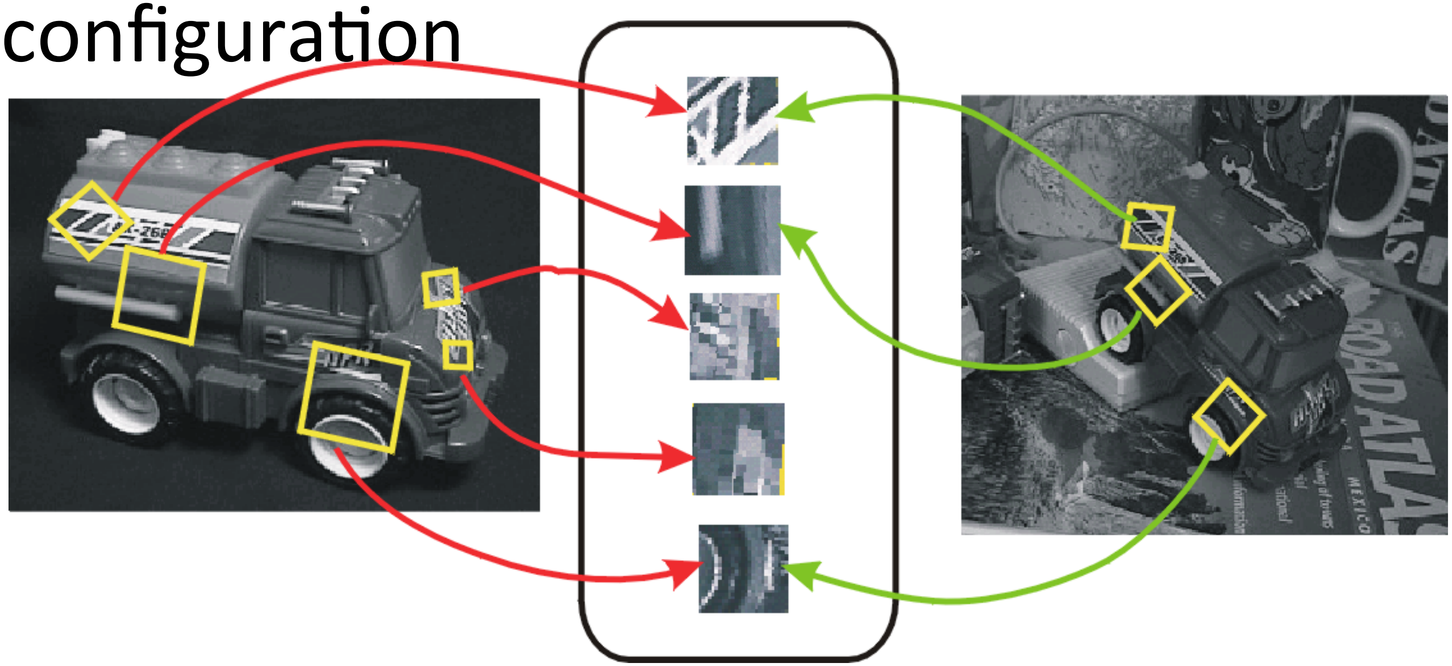


1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

K. Grauman, B. Leibe

Keypoint-based instance recognition

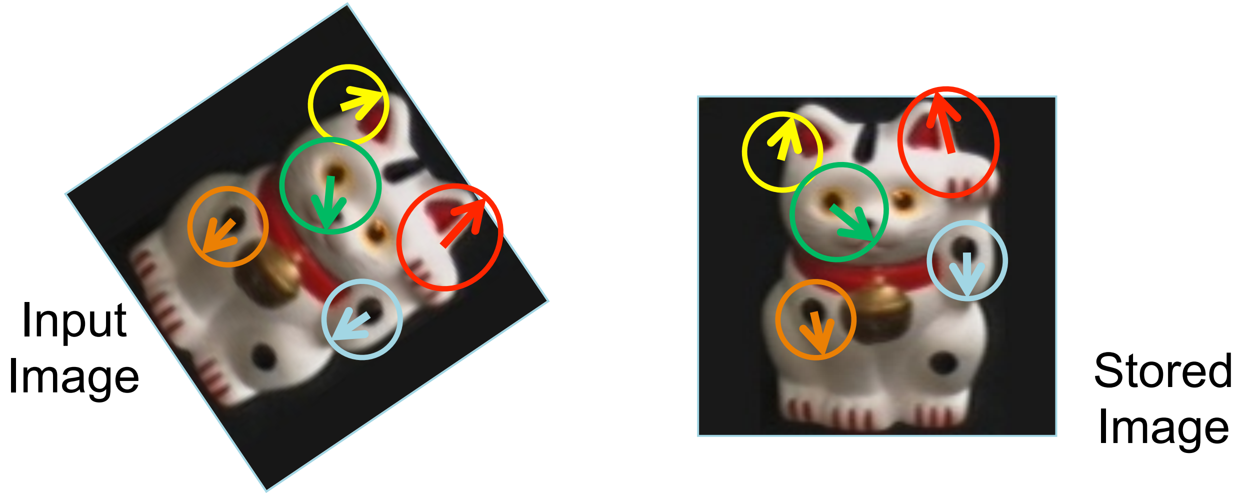
- Given two images and their keypoints (position, scale, orientation, descriptor)
- Goal: Verify if they belong to a consistent configuration



Local Features,
e.g. SIFT

Slide credit: David Lowe

Finding the objects (overview)



1. Match interest points from input image to database image
2. Matched points vote for rough position/orientation/scale of object
3. Find position/orientation/scales that have at least three votes
4. Compute affine registration and matches using iterative least squares with outlier check
5. Report object if there are at least T matched points

Matching Keypoints

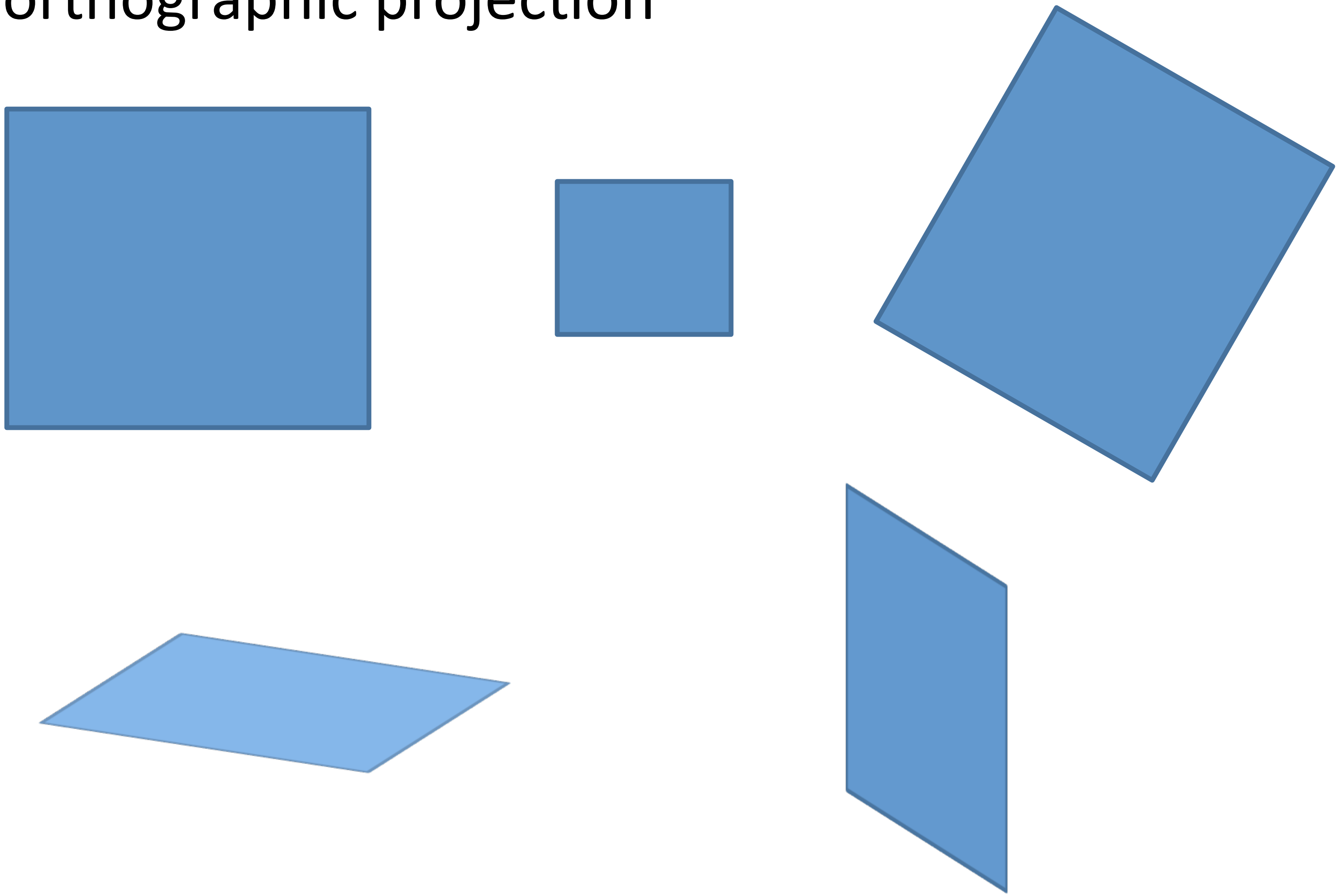
- Want to match keypoints between:
  1. Query image
  2. Stored image containing the object

- Given descriptor x_0, find two nearest neighbors x_1, x_2 with distances d_1, d_2

- x_1 matches x_0 if $d_1/d_2 < 0.8$
  – This gets rid of 90% false matches, 5% of true matches in Lowe's study

Affine Object Model

- Accounts for 3D rotation of a surface under orthographic projection



Affine Object Model

- Accounts for 3D rotation of a surface under orthographic projection

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ & & \vdots & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ \vdots \end{bmatrix}$$

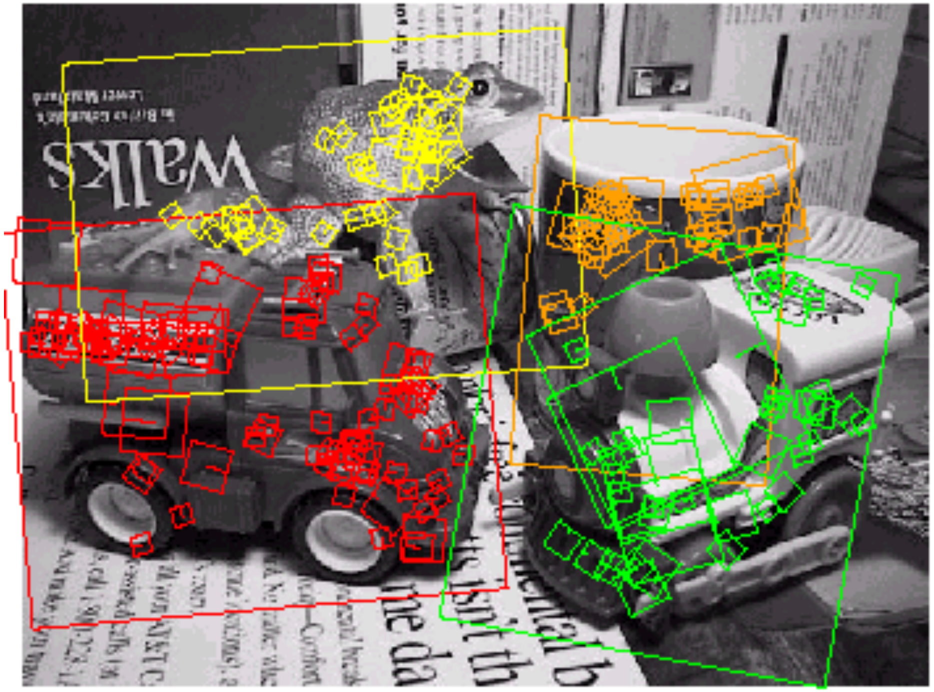
$$\mathbf{x} = [\mathbf{A}^T \mathbf{A}]^{-1} \mathbf{A}^T \mathbf{b}$$

What is the minimum number of matched points that we need?

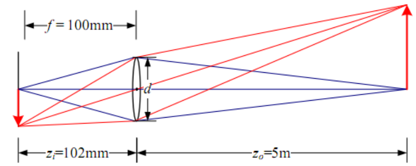
Finding the objects (SIFT, Lowe 2004)

1. Match interest points from input image to database image
2. Get location/scale/orientation using Hough voting
   - In training, each point has known position/scale/orientation wrt whole object
   - Matched points vote for the position, scale, and orientation of the entire object
   - Bins for x, y, scale, orientation
     - Wide bins (0.25 object length in position, 2x scale, 30 degrees orientation)
     - Vote for two closest bin centers in each direction (16 votes total)
3. Geometric verification
   - For each bin with at least 3 keypoints
   - Iterate between least squares fit and checking for inliers and outliers
4. Report object if > T inliers (T is typically 3, can be computed to match some probabilistic threshold)

Examples of recognized objects



Course Outline



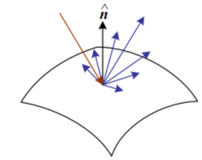
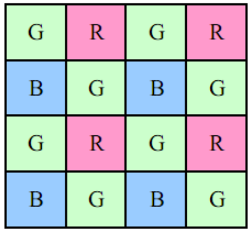


Image Formation and Processing

Light, Shape and Color

The Pin-hole Camera Model, The Digital Camera

Linear filtering, Template Matching, Image Pyramids



Feature Detection and Matching

Edge Detection, Interest Points: Corners and Blobs

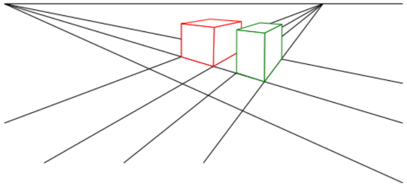
Local Image Descriptors

Feature Matching and Hough Transform

Multiple Views and Motion

Geometric Transformations, Camera Calibration

Feature Tracking , Stereo Vision



Segmentation and Grouping

Segmentation by Clustering, Region Merging and Growing

Advanced Methods Overview: Active Contours, Level-Sets, Graph-Theoretic Methods

Detection and Recognition

Problems and Architectures Overview

Statistical Classifiers, Bag-of-Words Model, Detection by Sliding Windows

Resources

Books

R. Szeliski, Computer Vision: Algorithms and Applications, 2010 – *available online*

D. A. Forsyth and J. Ponce, Computer Vision: A Modern Approach, 2003

L. G. Shapiro and G. C. Stockman, Computer Vision, 2001

Web

CVonline: The Evolving, Distributed, Non-Proprietary, On-Line Compendium of Computer Vision

http://homepages.inf.ed.ac.uk/rbf/CVonline/

Dictionary of Computer Vision and Image Processing

http://homepages.inf.ed.ac.uk/rbf/CVDICT/

Computer Vision Online

http://www.computervisiononline.com/

Programming

Development environments/languages: Matlab, Python and C/C++

Toolboxes and APIs: OpenCV, VLFeat Matlab Toolbox, Piotr's Computer Vision Matlab Toolbox, EasyCamCalib Software, FLANN, Point Cloud Library PCL, LibSVM, Camera Calibration Toolbox for Matlab