

i s t a n b u l t e c h n i c a l u n i v e r s i t y
a r c h i t e c t u r a l d e s i g n c o m p u t i n g g r a d u a t e p r o g r a m
dads 2011: a springtime venture

The Digital Shape or...

Mind the Gap Reloaded!

Lecture III

Implications of the Parallel Postulate

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Historical Timeline



Meanwhile; Indians and Chinese have been building their own mathematics ...
and geometry!

The Parallel Postulate

IF two straight lines in a plane are crossed by another straight line (called the transversal) and the interior angles between the two lines and the transversal, lying on one side of the transversal add up to less than two right angles;

THEN on that side of the transversal, the two lines extended will intersect.

During like 2000 years, people tried to prove this axiom using the first four...

None of the attempts were successful!

Equivalent Formulations

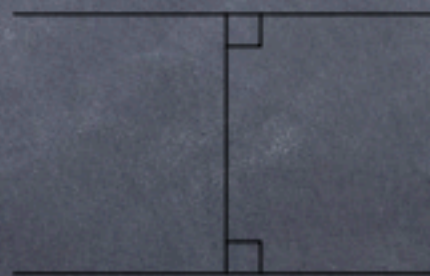
- There is at most one line that can be drawn parallel to another given one by an external point.
[Playfair]
- The sum of the angles in a triangle is exactly 180 degrees.
- Parallel (non-intersecting) lines remain at a constant distance from each other.
- Two non-parallel lines intersect each other at exactly one point.

Euclidean vs Non-Euclidean

- Geometries where the Parallel Postulate doesn't hold are non-Euclidean.
- Thus, the violation of the Parallel Postulate can be used as a diagnostic tool (to understand if we're in a Euclidean space or not).



Hyperbolic



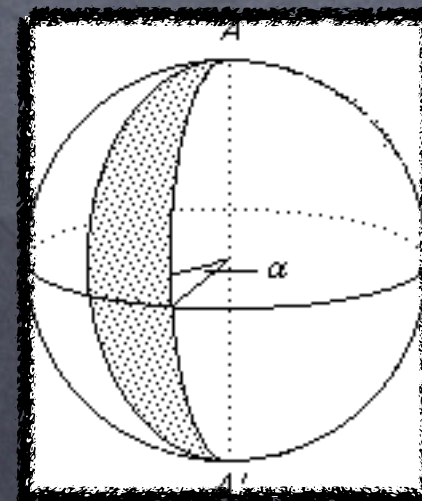
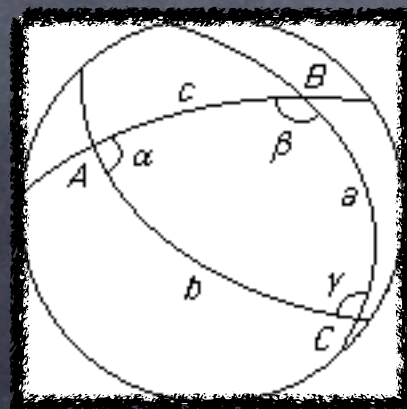
Euclidean



Elliptic

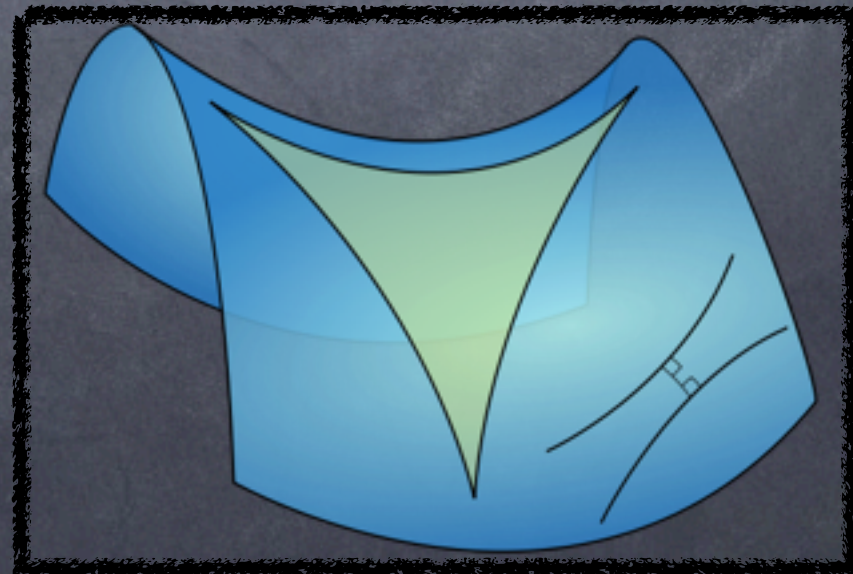
Euclidean vs Non-Euclidean

- **Elliptic** Geometry (The one of a **sphere**)
 - All lines (great circles) **intersect at exactly two points** (antipodal points).
 - “Parallel” lines **curve toward** each other.
 - Sum of the interior angles in a triangle is **greater** than 180 degrees.



Euclidean vs Non-Euclidean

- **Hyperbolic** Geometry (The one of a **saddle**)
 - Given a line l and a point P not on that line, there are **infinitely many** lines through P not intersecting l
 - Lines **curve away** each other.
 - Sum of the interior angles in a triangle is **less** than 180 degrees.



Moral of the Story

- The Parallel Postulate is not provable and its presence is necessary for a consistent (Euclidean) theory of the plane
- The opposite of the postulate does not lead to a **logical** contradiction (those encountered for the Euclidean plane are **observational**)
- Thus, spaces where the opposite is true are legitimate: these are **non-Euclidean** spaces.