Research Article
Density-Based 3D Shape Descriptors

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We propose a novel probabilistic framework for the extraction of density-based 3D shape descriptors using kernel density estimation. Our descriptors are derived from the probability density functions (pdf) of local surface features characterizing the 3D object geometry. Assuming that the shape of the 3D object is represented as a mesh consisting of triangles with arbitrary size and shape, we provide efficient means to approximate the moments of geometric features on a triangle basis. Our framework produces a number of 3D shape descriptors that prove to be quite discriminative in retrieval applications. We test our descriptors and compare them with several other histogram-based methods on two 3D model databases, Princeton Shape Benchmark and Sculpteur, which are fundamentally different in semantic content and mesh quality. Experimental results show that our methodology not only improves the performance of existing descriptors, but also provides a rigorous framework to advance and to test new ones.

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1. INTRODUCTION

The use of 3D models is becoming increasingly more commonplace with their distribution on the Internet and with the availability of 3D scanners. Many fields are focused on 3D object models: computer graphics, computer-aided design, medical imaging, molecular analysis, cultural heritage in virtual environments, movie industry, military target detection, or industrial quality control to name a few. Efficient organization and access to these databases demand effective tools for indexing, categorization, classification, and representation of 3D objects. All these database activities hinge on the development of 3D object similarity measures. There are two paradigms for 3D object database operations and design of similarity measures, namely, the feature vector approach and the nonfeature vector approach [1, 2]. The feature vector paradigm aims at obtaining numerical values of certain shape descriptors and measuring the distances between these vectors. A typical example of nonfeature-based approach is to describe the object as a graph and then use graph similarity metrics. In this work, we follow the feature vector paradigm, and furthermore we limit our scope to the subclass of histogram-based descriptors.

Representations used for shape matching are often referred to as 3D shape descriptors and they usually differ substantially from those intended for 3D object rendering and visualization [3]. Shape descriptors aim at encoding geometrical and topological properties of an object in a discriminative and compact manner. The diversity of shape descriptors range from 3D moments to shape distributions, from spherical harmonics to ray-based sampling, from point clouds to voxelized volume transforms [1, 2, 4–7]. In this work, inspired from histogram-based 3D shape descriptors [8–12], we propose a density-based approach that applies to local geometrical features of arbitrary dimension. Our interest in histogram-based 3D shape descriptors stems from their generality and their simplicity. They are global descriptors based on sets of local measurements and they have been shown to be effective in classifying shapes into broad categories [2]. Our objective is to show that, in addition to their categorization capability, they have also satisfactory retrieval performance.

Any histogram-based 3D shape descriptor must face the problem of estimating the histogram from any given mesh composed of triangles usually with arbitrary forms and sizes. In the previous histogram-based approaches, the surface samples are either chosen as the centers of gravity of the triangles or obtained by randomly sampling several points from the surface. A single sample from each triangle may not adequately represent the mesh. The random sampling of the
surface may compensate for the nonuniform distribution of triangles, provided that a sizeable number of surface points is taken. Although the random sampling approach proves to be useful for computing histograms of scalar features [10], it is not practical in the multidimensional case due to the curse of dimensionality: the number of samples required to fill in the multivariate histogram bins increases exponentially as dimensionality increases [13], resulting in a significant extra computational load which is not affordable for most applications such as retrieval.

Our density-based framework makes a more effective use of each triangle and also takes care of the nonuniformity of their areas and orientations without resorting to expensive random sampling. First, we do not use samples but exploit the information in the whole triangle area using an integration scheme, as described in Section 3.3. Second, we resort to nonparametric kernel density estimation (KDE) with rule-based bandwidth parameter assignment [13, 14]. In other words, local geometric information emanating from each mesh triangle contributes to the geometric feature density by the intermediary of a kernel. Thus local evidences about surface shape are accumulated at targeted density points to result in a global shape description. Third, we use a Gaussian kernel. Since the Gaussian density is completely determined by its first two moments, we only need to estimate the mean and the variance of the feature for each triangle. For certain cases, these moments can be approximated very accurately by making use of the geometry of a triangle in 3D space. The choice of Gaussian kernel brings in the additional advantage of alleviating the computational burden of calculating large sums of Gaussians, as occur in the proposed set of descriptors, by enabling the use of the efficient fast Gauss transform (FGT) [15, 16]. Thus the main contribution of our work is to propose an analytical framework for the extraction of 3D descriptors from local surface features that characterize the object geometry. This framework computes probability densities of local features instead of their conventional histograms. Here, we interpret histograms and densities in a broad sense: any descriptor that uses an accumulator scheme of measured quantities qualifies as a histogram-based descriptor. As a byproduct, we also introduce some novel local features.

The rest of the paper is structured as follows. In Section 2, we provide an overview of histogram-based 3D shape descriptors. Section 3 introduces the local geometric features we have considered and describes the KDE-based computational framework. In Section 4, we illustrate the retrieval performance of our method in comparison to other equivalent or similar histogram-based descriptors [8–12]. In Section 5, we draw conclusions and discuss further directions in density-based 3D shape descriptors.

2. PREVIOUS WORK ON 3D SHAPE DESCRIPTORS

There are two main paradigms of 3D shape description, namely, graph-based and vector-based. Graph-based representations are more elaborate and complex, harder to obtain, but represent shape properties in a more faithful and intuitive manner. Shock graphs [17], multiresolution Reeb graphs [6, 18, 19], and skeletal graphs [20] are methods that fall in this category. However, they do not generalize easily and hence they are not very convenient to use in unsupervised learning, for example, to search for natural shape classes in a database. Vector-based representations, on the other hand, are more easily computed. Although they do not necessarily conduce to plausible topological visualizations, they can be naturally employed in both supervised and unsupervised classification tasks. Typical vector-based representations are extended Gaussian images [8, 9], cord and angle histograms [11], 3D shape histograms [21], spherical harmonics [7, 22–24], and shape distributions [10]. In this work, we are exclusively interested in histogram-based 3D shape descriptors that constitute a particular branch of vector-based representations. In the following, we provide a brief overview of histogram-based descriptors. References [1, 2, 4] provide also excellent surveys.

In [11], Paquet and Rioux present cord and angle histograms for matching 3D objects. A “cord,” which is actually a ray, joins the barycenter of the mesh with a triangle center. The histograms of the length and of the angles of these rays (with respect to a reference frame) are used as the 3D shape descriptors. Although automatic determination of a canonical reference frame for 3D meshes is still not totally solved [7], the common practice is to obtain the eigendecomposition of the covariance matrix of the surface points. The covariance matrix itself can be computed using the mesh vertices, the triangle centers, or in a “continuous” way as described in [7]. The resulting eigenvectors, which are the orthogonal directions along which the mesh has maximal spread, are taken as a reference frame. Notice that the eigendirections may not necessarily correspond to the “natural” pose of the object; however, they can serve as a canonical reference frame. In conclusion, Paquet and Rioux [11] consider the shape descriptors consisting of the ray length and the relative ray angles with respect to the largest two eigenvectors. One shortcoming of all such approaches that reduce the triangles to their center points is that they do not take into consideration the size and shape of the mesh triangles. First, because triangles of any size have equal weight in the final shape distribution; second, because the triangle shapes can be arbitrary, so that the center may not represent adequately the impact of the triangle on the shape distribution.

In the shape distributions approach, Osada et al. [10] use a collection of shape functions, which are geometrical quantities estimated by a random sampling of the surface of the 3D object. Their shape functions are defined as the distance of surface points to the center of mass of the model (D1), the distance between two surface points (D2), the area of the triangle defined by three surface points (D3), the volume of the tetrahedron defined by four surface points (D4), and so on. The descriptors of the object are then defined as the histograms of these shape functions. The randomization of the surface sampling process improves the estimation over Paquet and Rioux’s approach [11], since a more representative and dense set of surface points is used. Obviously, the histogram accuracy can be controlled with the sample size.
Ankerst et al. use shape histograms for the purpose of molecular surface analysis [21]. A shape histogram is defined by partitioning the 3D space into concentric shells and sectors around the center of mass of a 3D model. The histogram is constructed by accumulating the surface points in the bins (in the form of shells, sectors, or both) based on a nearest-neighbor rule. Ankerst et al. [21] illustrate the shortcomings of Euclidean distance to compare two shape histograms and make use of a Mahalanobis-like quadratic distance measure taking into account the distances between histogram bins.

Extended Gaussian images (EGI), introduced by Horn [8], form another class of histogram-based 3D shape descriptors. An EGI consists of a spherical histogram with bins indexed by \((\theta, \phi)\), where each bin corresponds to some quantum of the spherical azimuth and elevation angles \((\theta, \phi)\) in the range \(0 \leq \theta < 2\pi\) and \(0 \leq \phi < \pi\). The histogram bins accumulate the count of the spherical angles of the surface normal per triangle, usually weighted by the triangle area. Kang and Ikeuchi have extended the EGI approach by considering the angular normal \(\hat{n}\) of a surface point to the center of the 3D shape is a one-dimensional \((3DHT)\) as a histogram constructed by accumulating surface points over planes in 3D space. Each triangle of the mesh contributes to each plane with a weight equal to the projected area of the triangle on the plane but only if the scalar product between their normals is higher than a given threshold. Although we have not encountered in the literature a direct comparison between 3DHT and EGI, 3DHT can be considered as a generalized version of EGI, where concentric spherical shells of different radii are constructed around the object’s center of mass. One can consequently conjecture that the 3DHT descriptor captures the shape information better than the EGI descriptor, as will be shown experimentally in Section 4.

An important property of a 3D shape descriptor is its invariance to similarity transformations, that is, translation (T), rotation (R), and scale (S) [1, 2, 4, 7]. In Table 1, we summarize invariance properties of the histogram-based shape descriptors discussed above.

### Table 1: Invariance properties of histogram-based 3D shape descriptors.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Translation invariance</th>
<th>Rotation invariance</th>
<th>Scale invariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cord histogram [11]</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Angle histogram [11]</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>D1-distribution [10]</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>D2-distribution [10]</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Shape histogram (shells) [21]</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Shape histogram (sectors) [21]</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>EGI [8]</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>CEGI [9]</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>3DHT [12]</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

3. THE PROPOSED FRAMEWORK FOR DENSITY-BASED DESCRIPTORS

3.1. Local geometric features

We assume that each 3D shape is represented as a triangular mesh and that its center of mass coincides with the origin of the coordinate system. In what follows, capital italic letter \(P\) stands for a point in 3D, a small case boldface letter \(p\) for its vector representation, \(\hat{n}_p = (\hat{n}_{px}, \hat{n}_{py}, \hat{n}_{pz})\) for the unit surface normal vector at \(P\) when \(P\) belongs to a surface \(\mathcal{M} \subset \mathbb{R}^3\), and \((\cdot, \cdot)\) for the usual dot product.

We define a local geometric feature as a mapping from the points of a surface \(\mathcal{M} \subset \mathbb{R}^3\) into a \(d\)-dimensional space, generally a subspace of \(\mathbb{R}^d\). Each dimension of this space corresponds to a specific geometric property that can be calculated at each point of the surface. For example, the distance of a surface point to the center of the 3D shape is a one-dimensional \((d = 1)\) geometric feature, while the mesh triangle normal \(\hat{n}_p\) is a three-dimensional feature vector \((d = 3)\). In this work, we consider three different multidimensional local geometric features that we describe in the sequel.

The radial feature \(S_r\) at a point \(P\) is a 4-tuple defined as:

\[
S_r(P) \triangleq (r_P, \hat{r}_{px}, \hat{r}_{py}, \hat{r}_{pz}) \quad \text{with} \quad \hat{r}_{px} \triangleq \frac{px}{r_P}, \quad \hat{r}_{py} \triangleq \frac{py}{r_P}, \quad \hat{r}_{pz} \triangleq \frac{pz}{r_P}.
\]

Accordingly, \(S_r\) consists of a magnitude component \(r_P\) measuring the distance of the point \(P\) to the origin, and a direction component \(\hat{r}_P = (\hat{r}_{px}, \hat{r}_{py}, \hat{r}_{pz})\) that gives the orientation of the point \(P\) (see Figure 1). Observe that we can write \(S_r\) also as \(S_r(P) = (r_P, \hat{r}_P)\). The direction component \(\hat{r}_P\) is a three-dimensional vector with unit norm; hence it lies on the unit sphere.
Given a set of observations \( \{ s_k \}_{k=1}^K \) for a random variable (scalar or vector) \( S \), the kernel approach to estimate the probability density of \( S \) is formulated in its most general form as

\[
f_s(s) = \sum_{k=1}^{K} w_k | H_k |^{-1} \mathcal{K}(H_k^{-1}(s - s_k)),
\]

where \( \mathcal{K} : \mathbb{R}^d \rightarrow \mathbb{R} \) is a kernel function, \( H_k \) is a \( d \times d \) matrix composed of a set of design parameters called bandwidth parameters (smoothing parameters or scale parameters) for the \( k \)th observation, and \( w_k \) is the importance weight associated with the \( k \)th observation. The contribution of each data point \( s_k \) to the density function \( f_s(s) \) at a target point \( s \) is computed through the kernel function \( \mathcal{K} \) scaled by the matrix \( H_k \) and the weight \( w_k \). Thus KDE involves a data set \( \{ s_k \}_{k=1}^K \) with the associated set of importance weights \( \{ w_k \}_{k=1}^K \), the choice of a kernel function \( \mathcal{K} \) and the setting of bandwidth parameters \( \{ H_k \}_{k=1}^K \).

We compute the probability density values of a certain local geometric feature \( S \) from a set of observations \( \{ s_k \}_{k=1}^K \). We assume that the 3D shape is represented as a triangular mesh consisting of \( K \) triangles. Thus we can obtain an observation \( s_k \) from each of the triangles in the mesh, as will be explained in Section 3.3. Since, in general, the mesh is made up of nonuniformly sized triangles, the data should be weighted accordingly. A natural choice for the importance weight \( w_k \) of a data point \( s_k \) is the ratio of the \( k \)th triangle area to the total surface area, yielding \( \sum_{k=1}^{K} w_k = 1 \). It is known that the particular functional form of the kernel does not significantly affect the accuracy of the estimator [14]. The Gaussian kernel has become a popular choice, first because it lends itself more easily to asymptotic error analysis [14]; and second, for the existence of efficient algorithms to calculate large sums of Gaussians, as the fast Gauss transform (FGT) already mentioned in the introduction [15, 16]. Actually, FGT is the dominant reason why we choose the Gaussian kernel since computational efficiency is an important requirement for 3D object retrieval [1, 2] (see Section 3.6 for details).

The setting of the bandwidth parameters \( \{ H_k \}_{k=1}^K \) is critical for an accurate kernel density estimation [14, 25]. For the Gaussian kernel, the bandwidth matrix \( H_k \) simply corresponds to the feature covariance matrix. For setting/estimating the bandwidth parameters, there exist several guidelines and computational methods with varying complexity [14, 25]. We discuss different alternatives in Section 3.4. The probability density function \( f_s(s) \), when computed over predefined target points using (4), results in the shape descriptor sought for a given triangular mesh. The methodology that we employ to choose the target points for each specific feature is explained in Section 3.5.

### Table 2: Local geometric features and their invariance properties (assuming that the barycenter of the surface \( \mathcal{M} \) is at the origin).

<table>
<thead>
<tr>
<th>Feature</th>
<th>Component-wise invariance</th>
<th>Overall invariance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial ( S_r )</td>
<td>Magnitude ( r_P ): rotation</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>Direction ( r_P ): scale</td>
<td></td>
</tr>
<tr>
<td>Tangent plane ( S_t )</td>
<td>Magnitude ( d_P ): rotation</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>Direction ( n_P ): scale</td>
<td></td>
</tr>
<tr>
<td>Cross-product ( S_c )</td>
<td>Magnitude ( r_P ): rotation</td>
<td>None</td>
</tr>
<tr>
<td></td>
<td>Direction ( c_P ): scale</td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: Radial and normal directions of a surface point.
3.3. Feature calculation

Given a $d$-dimensional local feature $S = (S_1, \ldots, S_d)$, the observation $s_k$ can be obtained from the mesh triangle $T_k$ by evaluating the value of $S$ at the barycenter of the triangle. However, the mesh triangles having in general arbitrary shapes, the feature value at the barycenter may not be the most representative one. The shape of the triangle should be in some way taken into account in order to reflect the local feature characteristics more faithfully. The expected value of the local feature $E[S \mid T]$ over the triangle $T$ is more informative than the feature value only sampled at a single point, the barycenter of the triangle.

Consider $T$ as an arbitrary triangle in 3D space with vertices $A$, $B$, and $C$ represented by $p_A$, $p_B$, and $p_C$, respectively, (see Figure 2). By noting $e_1 = p_B - p_A$ and $e_2 = p_C - p_A$, we can obtain a parametric representation for a point $P$ inside the triangle $T$ as $p = p_A + xe_1 + ye_2$, where the two parameters $x$ and $y$ satisfy the constraints $x, y \geq 0$ and $x + y \leq 1$. We assume that the point $P$ is uniformly distributed inside the triangle $T$. Thus, the expected value of the $i$th component of $S$, denoted by $E[S_i \mid T]$, is given by

$$E[S_i \mid T] = \int_{\Omega} S_i(x, y) f(x, y) dx dy, \quad i = 1, \ldots, d,$$

where $S_i(x, y)$ is the feature value at $(x, y)$ and $f(x, y)$ is the probability density function of the pair $(x, y)$ over the domain $\Omega = \{(x, y) : x, y \geq 0, x + y \leq 1\}$. Accordingly, $f(x, y) = 2$ when $(x, y) \in \Omega$ or zero otherwise. The integration is performed over the domain $\Omega$. To approximate (5), we apply Simpson’s 1/3 numerical integration formula [26]. We avoid the arbitrariness in vertex labeling by considering the three permutations of the labels $A$, $B$, and $C$. This yields us three approximations, which are in turn averaged to yield

$$E[S_i \mid T] \approx \left( \frac{1}{27} \right) (S_i(p_A) + S_i(p_B) + S_i(p_C))$$

$$+ \left( \frac{4}{27} \right) \left( S_i \left( \frac{p_A + p_B}{2} \right) + S_i \left( \frac{p_A + p_C}{2} \right) \right)$$

$$+ \left( \frac{4}{27} \right) \left( S_i \left( \frac{p_B + p_C}{2} \right) \right),$$

(6)

Equation (6) boils down to take a weighted average of feature values calculated at 9 points on the triangle.

3.4. Bandwidth selection

There are three levels of analysis at which the parameters in the bandwidth matrix $H_k$ involved in KDE can be chosen (see (4) in Section 3.2).

(1) Triangle level: this option allows a distinct bandwidth parameter for each triangle in the mesh. In principle, this choice is very flexible since it does not make any assumptions about the shape of the kernel function and hence about the shape of the 4th triangle. In general, finding a KDE bandwidth matrix specific to each observation is a difficult problem [25]. For the Gaussian kernel, however, estimation of the bandwidth matrix $H_k$ reduces to the estimation of the feature covariance matrix. The moment formula in (5) and its numerical approximation in (6) can directly be used for moments of any order. For example, the $(i, j)$th component $h_{ij}$ of $H$ is computed by

$$h_{ij} = \int_{\Omega} S_i(x, y) S_j(x, y) f(x, y) dx dy$$

$$- \int_{\Omega} S_i(x, y) f(x, y) dx dy$$

$$\times \int_{\Omega} S_j(x, y) f(x, y) dx dy, \quad i, j = 1, \ldots, d.$$

(7)

(2) Mesh level: the second option is to use a fixed bandwidth matrix for all triangles in a given mesh, but different bandwidths for different meshes. In this case, the bandwidth matrix for a given feature can be obtained from its observations using Scott’s rule of thumb [14]: $H_{Scott} = (\sum_k w_k^2)^{1/(d+4)} \hat{C}^{1/2}$, where $d$ is the dimension of the feature, $\hat{C}$ is the estimate of the feature covariance matrix, and $w_k$ is the weight associated to each observation. Scott’s rule of thumb is proven to provide the optimal bandwidth in terms of estimation error when the kernel function and the unknown density are both Gaussian. Although, there is no guarantee that feature distributions to be Gaussian, Scott’s rule of thumb is still used for its simplicity.
and $S_t$ is discussed in Section 4.2. The direction components are uniformly quantized in the interval $[0, \pi]$, achieved by subdividing an octahedron once (left: 32 points) and twice (right: 128 points).

Figure 3: Distribution of target points over the unit-sphere, obtained by subdividing an octahedron once (left: 32 points) and twice (right: 128 points).

(3) Database level: in the last option, the bandwidth parameter is fixed for all triangles and meshes, that is, $H_k = H$. Setting the bandwidth at database level has the implicit effect of smoothing the resulting densities. In this case, we estimate the bandwidth parameters from a representative subset of the database by averaging the Scott bandwidth matrices over the selected meshes.

### 3.5. Choice of the targets

Targets are defined as the points at which the feature density functions are explicitly calculated. The density values computed at these targets constitute the 3D shape feature vector. Selection of target points must result in parsimonious yet discriminative descriptors. For single-dimensional features, it suffices to uniformly sample the density function within its dynamic range. However, the multidimensional features, $S_r$, $S_t$, and $S_c$, which consist of magnitude and direction components, require more attention. We denote the target size by $N_{mag}$ for the magnitude component and by $N_{dir}$ for the direction component. The target points for these multidimensional features are then obtained by the Cartesian product of the two sets, yielding an overall target set size of $N = N_{mag} \times N_{dir}$. The magnitude components of $S_r$ and $S_t$ are uniformly quantized in the interval $[0, r_{\max}]$, while those of $S_c$ in the $[0, d_{t,\max}]$ interval. The setting of $r_{\max}$ and $d_{t,\max}$ is discussed in Section 4.2. The direction components of $S_r$ and $S_t$ features, namely, $\hat{r}_F$ and $\hat{n}_F$, lie on the unit sphere. To complete the design of target points, following [12], we consider an octahedron circumscribed by the unit sphere and we subdivide each of its 8 triangles into four, twice, by radially projecting back the subdivided triangles to the surface of the sphere. As targets of the direction components of $S_r$ and $S_t$, we select the barycenters of the resulting 128 triangles, 16 per each of the 8 faces of the octahedron. This leads to a uniform partitioning of the sphere, as shown in Figure 3.

The $S_c$ feature has a direction component $c_p$ with non-unit norm, which lies within the unit ball. For the target set of the direction component $c_p$, we thus similarly consider octahedra, but circumscribed by spheres of various radii. We take four such octahedra within spheres of radial length 0.25, 0.5, 0.75, and 1. We subdivide the two inner octahedra once, each yielding 32 targets, and the two outer octahedra twice, each yielding 128 targets. This gives a total of $N_{dir} = 320$ regularly spaced targets for the $c_p$-component of the $S_c$ feature. The inner spheres have sparser targets to balance out the target densities of the outer spheres.

### 3.6. Computational complexity of KDE

The computational complexity of KDE using directly (4) is $O(KN)$, where $K$ is the number of observations (the number of triangles in our case) and $N$ is the number of density evaluation points, that is, targets. For applications such as content-based retrieval, the $O(KN)$-complexity is prohibitive. To give an example, on a Pentium 4 PC (2.4 GHz CPU, 2 GB RAM) and for a mesh of 130,000 triangles, the direct evaluation of the $S_r$-descriptor (1024-point pdf) takes 125 seconds. However, when the kernel function in (4) is chosen as Gaussian, we can use the fast Gauss transform (FGT) [15, 16] to reduce the computational complexity by two orders of magnitude. For example, with FGT, the $S_r$-descriptor computation takes only 2.5 seconds. FGT is an approximation scheme enabling the calculation of large sums of Gaussians within reasonable accuracy and reducing the complexity down to $O(K + N)$. In our 3D shape description system, we have used an improved version of FGT implemented by Yang et al. [16].

For the sake of completeness, we provide the conceptual guidelines of the FGT algorithm (see [15, 16] for mathematical and implementation details). FGT is a special case of the more general fast multipole method [15], which trades off computational simplicity for acceptable loss of accuracy. The basic idea is to cluster the data points and target points using appropriate data structures and to replace the large sums with smaller ones that are equivalent up to a given precision. In the case of FGT, each exponential in the sum is shifted and expanded into a truncated Hermite series in $O(K)$ operations. The gain in complexity is achieved by avoiding the computation of every Gaussian at every evaluation point unlike the direct approach, which has $O(KN)$-complexity. The accuracy can be controlled by the truncation order. Truncated Hermite series are constructed about a small number of cluster centers formed by target points; the series are shifted to target cluster centers, and then evaluated at $N$ targets in $O(N)$ operations. Since the two sets of operations are disjoint, the total complexity of FGT becomes $O(K + N)$.

### 3.7. Flow diagram of the algorithm

We summarize below the proposed algorithm to obtain a density-based 3D shape descriptor.

1. For a chosen local feature $S$, specify a set of targets $t_n$, $n = 1, \ldots, N$.
2. Normalize the 3D triangular mesh $\mathcal{M} = \bigcup_{k=1}^{K} T_k$ according to the invariance requirements of $S$.
3. For each mesh triangle $T_k$, calculate its feature value $s_k$ using (6) and its weight $w_k$. 


4. EXPERIMENTAL RESULTS

In this section, we illustrate the performance of the proposed shape descriptors in 3D retrieval applications. When a query model is presented to the 3D object database, its descriptor is calculated and then compared to all the stored descriptors using a distance function. The outcome is a set of database models sorted in increasing distance. The models at the top of the list are expected to resemble the queried model more than those at the bottom of the list.

We have experimented on two different 3D model databases: the Princeton Shape Benchmark (PSB) [5] and the Sculpteur Database (SCUdb) [6, 27]. Both databases consist of objects described as triangular meshes, though they differ substantially in terms of content and mesh quality. PSB is a publicly available database containing a total of 1814 synthesis models, categorized into general classes such as animals, humans, plants, household objects, tools, vehicles, buildings, and so forth. An important feature of the database is the availability of two equally sized sets. One of them is a training set (90 classes) reserved for tuning the parameters involved in the computation of a particular shape descriptor, and the other for testing purposes (92 classes). By contrast, SCUdb is a private database containing over 800 models corresponding mostly to scanned archeological objects residing in museums [6, 27]. Presently, 513 of the models are classified into 53 categories with comparable set populations, which include utensils of ancient times (e.g., amphorae, vases, bottles, etc.), pavements, and artistic objects such as human statues (parts or as a whole), figurines, and moulds. The database has been augmented by artificially generated 3D objects such as spheres, tori, cubes, or cones in order to build a set of simple well-controlled classes. The meshes in SCUdb are highly detailed and reliable in terms of connectivity and orientation of triangles. To give an idea of the significant differences between PSB and SCUdb, we can quote average mesh resolution figures. The average number of triangles in SCUdb and in PSB is 175250 and 7460, respectively, corresponding to a ratio of 23. In terms of vertices, SCUdb meshes contain 87670 vertices on the average while for PSB this number is 4220. Furthermore, the average triangular area relative to the total mesh area is 33 times smaller in SCUdb than in PSB.

4.1. Evaluation tools

The most commonly used statistics for measuring the performance of a shape descriptor in a content-based retrieval application are summarized below [5].

(i) Precision-recall curve

For a query $q$ that is a member of a certain class, Precision (vertical axis) is the ratio of the relevant matches $K_q$ (matches that are within the same class as the query) to the number of retrieved models $K_{ret}$, and Recall (horizontal axis) is the ratio of relevant matches $K_q$ to the size of the query class $C_q$:

$$\text{Precision} = \frac{K_q}{K_{ret}}, \quad \text{Recall} = \frac{K_q}{C_q}. \quad (8)$$
(ii) Nearest neighbor (NN)

The percentage of the first-closest matches that belong to the query class.

(iii) First-tier and second-tier

First-tier (FT) is the recall when the number of retrieved models is the same as the size of the query class and second-tier (ST) is the recall when the number of retrieved models is two times the size of the query class.

(iv) E-measure

This is a composite measure of the precision and recall for a fixed number of retrieved models, for example, 32, based on the intuition that a user of a search engine is more interested in the first page of query results than in later pages. E-measure is given by

\[
E = \frac{2}{1/\text{precision} + 1/\text{recall}}. 
\]

(v) Discounted cumulative gain

A statistic that weights correct results near the front of the list more than correct results later in the ranked list under the assumption that a user is less likely to consider elements near the end of the list. Specifically, the ranked list of retrieved objects is converted to a list \( L \), where an element \( L_k \) has value 1 if the \( k \)-th object in the ranked list is in the same class as the query and otherwise has value 0. Discounted cumulative gain \( \text{DCG}_k \) is then defined as

\[
\text{DCG}_k = \begin{cases} 
L_k, & k = 1, \\
\text{DCG}_{k-1} + \frac{L_k}{\log_2(k)}, & \text{otherwise}.
\end{cases}
\]

The final DCG score for a query \( q \) is obtained for \( k = K_{\text{max}} \), where \( K_{\text{max}} \) is the total number of objects in the database, and normalizing \( \text{DCG}_K \) by the maximum possible DCG that would be achieved if the first \( C_q \) retrieved elements were in the class of the query \( q \) (\( C_q \) is the size of the query class). Thus DCG reads as

\[
\text{DCG} = \frac{\text{DCG}_{K_{\text{max}}}}{1 + \sum_{k=2}^{\text{DCG}_{K_{\text{max}}}} (1/\log_2(k))}. 
\]

(vi) Normalized DCG

This is a very useful statistic based on averaged DCG values of a set of algorithms on a particular database. Normalized DCG (NDCG) gives the relative performance of an algorithm with respect to the other ones. A negative value means that the performance of the algorithm is below the average; similarly a positive value indicates above the average performance. Let \( \text{DCG}^{(A)} \) be the DCG of a certain algorithm \( A \) and let \( \text{DCG}^{(\text{avg})} \) be the average DCG values of a series of algorithms on the same database, then NDCG for the algorithm \( A \) is defined as

\[
\text{NDCG}^{(A)} = \frac{\text{DCG}^{(A)}}{\text{DCG}^{(\text{avg})}} - 1. 
\]

All these quantities are normalized within the range \([0, 1]\) (except NDCG) and higher values reflect better performance. In order to give the overall performance of a shape descriptor on a database, the values of a statistic for each query are averaged to yield a single performance figure. The retrieval statistics presented in the sequel are obtained using the utility software included in PSB [5].

4.2. Retrieval experiments

In all of our retrieval experiments, we use the Minkowski-\( l_1 \) distance measure to assess the similarity between descriptors since we have observed that this distance function gives better performance in most of the cases as compared to other distance measures such as \( l_2 \) or \( \chi^2 \). We apply the following normalization to all the meshes of the database to secure RST invariance of the features. For translation invariance, the object’s center of mass is translated to the origin. For scale invariance, the area-weighted average distance of surface points to the origin is set to unity. We have observed that, with this scaling operation, the frequency of the distance of a surface point to the mesh center exceeding 2 becomes negligible. This allows us to set empirical upper limits \( r_{\text{max}} \) and \( d_{\text{r, max}} \) to the magnitude components \( r_P \) and \( d_{\text{r, p}} \), respectively. Finally, to guarantee rotation and reflection invariance, we follow the “continuous” PCA approach of Vranic [7]. All the codes for our descriptors as well as for those proposed in the literature (cord and angle histograms [11], D1 and D2 shape distributions [10], EGI [8] and CEGI [9], 3DHT [12]) have been implemented in MATLAB 7.0 (R14) environment, using C MEX external interface for time-consuming jobs. For FGT, we have used the implementation provided by Yang et al. [16].

The acronyms of the descriptors we have experimented are listed in Tables 3 and 4. They will subsequently be used in graph annotations. The details about descriptor sizes are given in the corresponding sections.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Acronym</th>
<th>Size ( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cord and angle histograms [11]</td>
<td>CAH</td>
<td>( 4 \times 64 = 256 )</td>
</tr>
<tr>
<td>D1-distribution [10]</td>
<td>D1</td>
<td>64</td>
</tr>
<tr>
<td>D2-distribution [10]</td>
<td>D2</td>
<td>64</td>
</tr>
<tr>
<td>EGI [8]</td>
<td>EGI</td>
<td>128</td>
</tr>
<tr>
<td>3DHT [12]</td>
<td>3DHT</td>
<td>( 8 \times 128 = 1024 )</td>
</tr>
</tbody>
</table>

A is listed in Table 3: Histogram-based 3D shape descriptors and their sizes.
Table 4: Density-based 3D shape descriptors and their sizes.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>Acronym</th>
<th>Size $N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial ($S_r$) density</td>
<td>Sr</td>
<td>$8 \times 128 = 1024$</td>
</tr>
<tr>
<td>Tangent pl. ($S_t$) density</td>
<td>St</td>
<td>$8 \times 128 = 1024$</td>
</tr>
<tr>
<td>Cross-product ($S_c$) density</td>
<td>Sc</td>
<td>$8 \times 320 = 2560$</td>
</tr>
<tr>
<td>Normal ($S_n$) density</td>
<td>Sn</td>
<td>$128$</td>
</tr>
</tbody>
</table>

Univ. dens. of $S_r$ components $[S_{r1}, S_{r2}, S_{r3}, S_{r4}]$ $4 \times 64 = 256$

Univ. dens. of $S_t$ components $[S_{t1}, S_{t2}, S_{t3}, S_{t4}]$ $4 \times 64 = 256$

Table 5: DCG values for possible bandwidth selection strategies on PSB training meshes.

<table>
<thead>
<tr>
<th>Bandwidth setting</th>
<th>Sr</th>
<th>St</th>
<th>Sc</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle level</td>
<td>0.352</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Mesh level</td>
<td>0.511</td>
<td>0.514</td>
<td>0.499</td>
</tr>
<tr>
<td>Database level</td>
<td>0.541</td>
<td>0.567</td>
<td>0.543</td>
</tr>
</tbody>
</table>

separately be computed by univariate density estimation and then concatenated in a joint vector, as in the last two rows of Table 4. Let $A_1, A_2, \ldots, A_L$ denote $L$ generic (one- or multidimensional) features and let $f_{A_1}, f_{A_2}, \ldots, f_{A_L}$ denote the corresponding density-based descriptors with $N_1, N_2, \ldots, N_L$ components, respectively, ($N_i, i = 1, \ldots, L$ corresponds to the number of target points on which the density of feature $A_i$ has been evaluated or equivalently to the size of the vector $f_{A_i}$). Square bracketing $[A_1, A_2, \ldots, A_L]$ that appears in subsequent graphs and tables indicates the concatenation of the shape descriptors $[f_{A_1}, f_{A_2}, \ldots, f_{A_L}]$ resulting in a vector of size $N_1 + N_2 + \cdots + N_L$. For notational simplicity, we will refer to the descriptor $f_{A_1}$ consisting of the density vector as $A_1$-descriptor; similarly, $[A_1, A_2]$ will be the shorthand notation for the descriptor $[f_{A_1}, f_{A_2}]$. Note finally that the generic feature $A_i$ can be either a vector by construction or a scalar obtained by taking a component of some other multidimensional feature.

4.2.1. Impact of bandwidth selection

The KDE approach critically depends upon the judicious setting of the bandwidth parameters. We tested the triangle, mesh and database level alternatives presented in Section 3.4 on our multidimensional local features $S_r, S_t,$ and $S_c$ (the computationally expensive triangle-level setting was only tested for $S_r$). Since we have observed that the off-diagonal terms of the bandwidth matrices are negligible as compared to the diagonal terms, we use only diagonal bandwidth matrices $H = \text{diag}(h_1, \ldots, h_d)$. For the mesh level and database level, we apply the Scott’s rule-of-thumb. For the triangle level, we employ the KDE toolbox developed by Ihler [28] since the available FGT implementation does not allow a different bandwidth per triangle [16]. The KDE toolbox makes use of kd-trees and reduces the computational burden considerably, though not to the extent achieved by FGT. Table 5 compares the DCG scores obtained with Sr, St, and Sc-descriptors on the PSB training set. Figure 5 shows the precision-recall plots corresponding to mesh and database level settings for Sr and St-descriptors. We clearly observe that setting the bandwidth $H$ at database level is more advantageous as compared to triangle and mesh level settings.

Any further results reported are therefore for the database level setting of $H$. In Table 6, we provide the average Scott bandwidth values obtained from PSB training meshes for $S_r$, $S_t$, and $S_c$ features.

4.2.2. Univariate versus multivariate density-based descriptors

In this section, we compare the impact of combining descriptors on the retrieval performance. As discussed before,
Table 6: The average Scott bandwidth obtained from the PSB training meshes.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>$h_1$</th>
<th>$h_2$</th>
<th>$h_3$</th>
<th>$h_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sr</td>
<td>0.20</td>
<td>0.35</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>St</td>
<td>0.20</td>
<td>0.25</td>
<td>0.25</td>
<td>0.30</td>
</tr>
<tr>
<td>Sc</td>
<td>0.20</td>
<td>0.15</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

descriptors can be compounded either by concatenating univariate descriptors or by multivariate density estimation. One can conjecture that the multivariate descriptors, resulting from the joint density functions of features, are richer in information content since component-wise dependencies are also taken into account. On the other hand, univariate densities are much simpler to estimate and do not incur into dimensionality problems. In our experiments, each univariate density is evaluated at 64 target points. Accordingly, a 4-tuple concatenation, such as $[Sr1,Sr2,Sr3,Sr4]$, results in a descriptor of size $N = 4 \times 64 = 256$. For multivariate density descriptors $Sr$ and $St$, recall that $N_{dir} = 128$ and for $Sc$, $N_{dir} = 320$ (see Section 3.5). $N_{mag}$ being chosen equal to 8 in all cases, the size of the $Sr$ and $St$-descriptors is $N = 8 \times 128 = 1024$ and the size of the $Sc$-descriptor is $N = 8 \times 320 = 2560$. Figures 6 and 7 with Table 7 explicitly show that the multivariate density-based descriptors are superior to the descriptors obtained by the concatenation of univariate densities for all feature types on both databases.

4.2.3. Comparison of density-based descriptors with their histogram-based peers

One of the motivations of this work is to show that a considerable improvement in the retrieval performance can be obtained by more rigorous and accurate computation of shape distributions as compared to more practical ad hoc histogram approaches. Notice that we interpret the term “histogram-based descriptor” for any count-and-accumulate type of procedure. This way we can refer to analogous descriptors in the literature as histogram-based whenever they count-and-accumulate local information to obtain a global shape descriptor [8–12].

An interesting case in point is Cord and Angle Histograms (CAH) [11]. The features in CAH are identical to the individual scalar components $r_p, r_{px}, r_{py}$, and $r_{pz}$ of our $Sr$ feature up to a parameterization. In [11], the authors consider the length of a cord (corresponding to $r_p$) and the two angles between a cord and the first two principal directions (corresponding to $r_{px}$ and $r_{py}$). Notice that in our parameterization of $Sr$, we consider the Cartesian coordinates rather than the angles. In order to compare with our $[Sr1,Sr2,Sr3,Sr4]$-descriptor, we implemented the CAH-descriptor by also considering the histogram of the angle with the third principal direction. The resulting CAH-descriptor is thus the concatenation of one cord length and three angle histograms. Each histogram consisting of 64 bins leads to a descriptor of total size $N = 4 \times 64 = 256$. $[Sr1,Sr2,Sr3,Sr4]$-descriptor, again of size 256, differs from CAH in three aspects: first, it uses a different parameterization of the angle (direction) components; second, the local feature values are calculated by (6) instead of using mere barycentric sampling; third, it employs KDE instead of histogram computation. In Figure 8, we provide the precision-recall curve corresponding to CAH and $[Sr1,Sr2,Sr3,Sr4]$ on PSB test set and on SCUdb. The respective DCG values are 0.434 and 0.501 for PSB, 0.681 and 0.698 for SCUdb, indicating the superior performance of our framework under identical feature sets. An additional improvement can be gained by estimating the joint density of $Sr$, leading to the $Sr$-descriptor. That is, in contrast to the concatenation of univariate densities, we directly use the joint density of $Sr$ as a descriptor. The DCG value
of the Sr-descriptor is 0.533 on PSB and 0.708 on SCUdb, one more step of improvement as compared to the concatenated univariate case [Sr1,Sr2,Sr3,Sr4] (DCG = 0.501 on PSB and DCG = 0.698 on SCUdb). Note that the performance improvement using our scheme is less impressive over SCUdb than over PSB. This can be explained by the fact that SCUdb meshes are much denser than PSB meshes in number of triangles. As the number of observations increases, the accuracies of the histogram method and KDE become comparable and both methods result in similar descriptors. This also indicates that the KDE methodology is especially appropriate for coarser mesh resolutions as in PSB.

A second instance of our framework outperforming its competitor is with the EGI-descriptor [2, 5, 8], which consists of binning the surface normals. The density of our Sn(P) = \(\hat{n}_P\) feature is equivalent to the EGI-descriptor. There can be different choices for binning surface normals, for example, by mapping the normal of a certain mesh triangle to the closest bin on the unit sphere and augmenting that bin by the relative area of the triangle. Such an approach requires a very densely discretized unit sphere and the resulting descriptor is not very efficient in terms of storage. In the present work, similarly to [12], we preferred the following implementation for the EGI-descriptor. First, 128 unit norm vectors \(\hat{n}_{bin,j}\), \(j = 1, \ldots, 128\), are obtained as histogram bin centers by octahedron subdivision, as described in Section 3.5. Then, the contribution of each triangle \(T_k\), \(k = 1, \ldots, K\), with normal vector \(\hat{n}_k\) to the \(n_{th}\) bin center is computed as \(w_k|\langle \hat{n}_k, \hat{n}_{bin,j}\rangle|\) if \(|\langle \hat{n}_k, \hat{n}_{bin,j}\rangle| \geq 0.7\) or otherwise as zero (recall that \(w_k\) is the relative area of the \(k_{th}\) triangle). The use of the absolute value is needed because some models as those in the PSB set cannot provide orientation information. The Sn-descriptor of the same size, that is, 128, achieves a superior DCG of 0.478 as compared to the DCG score of 0.438 for EGI on PSB (see Figure 9). For SCUdb, the DCG-performance differential is even more pronounced (DCG = 0.589 for Sn, DCG = 0.535 for EGI) noting that for low recall values (recall < 0.2), the EGI-descriptor is better than Sn (see Figure 9).

A third instance of comparison can be considered between our St-descriptor and the 3DHT-descriptor [12] since both of them use local tangent plane parameterization. The procedure for the 3DHT descriptor is carried out as follows. We first recall that the 3DHT-descriptor is a histogram constructed by accumulating mesh surface points over planes in 3D space. Each histogram bin corresponds to a plane \(\mathcal{P}_{ij}\) parameterized by its normal distance \(d_{i,j}\), \(i = 1, \ldots, N_{mag}\), to the origin and its normal direction \(\mathbf{n}_{bin,j}\), \(j = 1, \ldots, N_{dir}\). Clearly, there can be \(N_{mag} \times N_{dir}\) such planes and the resulting descriptor is of size \(N = N_{mag} \times N_{dir}\). We can obtain such a family of planes exactly as described in Section 3.5 and in [12]. In our experiments, we have used \(N_{mag} = 8\) distance bins sampled within the range \([0, 2]\) and \(N_{dir} = 128\) uniformly sampled normal directions. This results in a 3DHT descriptor of size \(N = 1024\). To construct the Hough array, one first takes a plane with normal direction \(\mathbf{n}_{bin,j}\), \(j = 1, \ldots, N_{dir}\), at each triangle barycenter \(\mathbf{m}_k\), \(k = 1, \ldots, K\),

### Table 7: Retrieval statistics for univariate and multivariate density-based descriptors.

<table>
<thead>
<tr>
<th>Descriptor</th>
<th>NN</th>
<th>FT</th>
<th>ST</th>
<th>E</th>
<th>DCG</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Sr1,Sr2,Sr3,Sr4]</td>
<td>0.436</td>
<td>0.222</td>
<td>0.306</td>
<td>0.180</td>
<td>0.501</td>
</tr>
<tr>
<td>Sr</td>
<td>0.499</td>
<td>0.260</td>
<td>0.343</td>
<td>0.201</td>
<td>0.533</td>
</tr>
<tr>
<td>[St1,St2,St3,St4]</td>
<td>0.451</td>
<td>0.250</td>
<td>0.348</td>
<td>0.202</td>
<td>0.533</td>
</tr>
<tr>
<td>St</td>
<td>0.523</td>
<td>0.267</td>
<td>0.364</td>
<td>0.210</td>
<td>0.543</td>
</tr>
<tr>
<td>[Sr1,Sr2,Sr3,Sr4]</td>
<td>0.701</td>
<td>0.430</td>
<td>0.555</td>
<td>0.314</td>
<td>0.698</td>
</tr>
<tr>
<td>Sr</td>
<td>0.745</td>
<td>0.452</td>
<td>0.568</td>
<td>0.323</td>
<td>0.709</td>
</tr>
<tr>
<td>[St1,St2,St3,St4]</td>
<td>0.632</td>
<td>0.400</td>
<td>0.520</td>
<td>0.298</td>
<td>0.662</td>
</tr>
<tr>
<td>St</td>
<td>0.754</td>
<td>0.473</td>
<td>0.575</td>
<td>0.324</td>
<td>0.712</td>
</tr>
</tbody>
</table>

![Figure 7: Precision-recall curves for [Sr1,Sr2,Sr3,Sr4] versus Sr (a) and [St1,St2,St3,St4] versus St on SCUdb (b).](image-url)
Recall
Precision
DCG = 0.434
DCG = 0.501
DCG = 0.533

CAH
[ Sr1, Sr2, Sr3, Sr4]
Sr

(a)
(b)

Figure 8: Precision-recall curves for CAH, [Sr1, Sr2, Sr3, Sr4] (concatenated) and Sr (joint) on PSB test set (a) and SCUdb (b).

Recall
Precision
DCG = 0.698
DCG = 0.681
DCG = 0.708

CAH
[ Sr1, Sr2, Sr3, Sr4]
Sr

(a)
(b)

Figure 9: Precision-recall curves for EGI and Sn on PSB test set (a) and SCUdb (b).

and then calculates the normal distance of the plane to the origin by $|\langle m_k, \hat{n}_{bin}, j \rangle|$. The resulting value is quantized to the closest $d_{it}, i = 1, \ldots, N_{mag}$, and then the bin corresponding to the plane $P_{ij}$ is augmented by $w_k |\langle \hat{n}_k, \hat{n}_{bin}, j \rangle|$ if $|\langle \hat{n}_k, \hat{n}_{bin}, j \rangle| \geq 0.7$ (the value of 0.7 is suggested by Zaharia and Prêteux [12] and we have also verified its performance-wise optimality). In Figure 10, we compare the St- and the 3DHT-descriptors in terms of precision-recall curves. On PSB, the St-descriptor yields a DCG of 0.543, a worse score against 0.577 of the 3DHT-descriptor. This can be attributed largely to the fact that the 3DHT-descriptor employs an implicit correction for normal orientations by the weighting scheme $w_k |\langle \hat{n}_k, \hat{n}_{bin}, j \rangle|$ according to which only normal direction $\hat{n}_k$ matters but not its orientation. Our St-descriptor does not make use of such a correction and considers the normal orientations as they are provided by the list of triangles in the mesh. Accordingly, we explain the negative performance gap between St and 3DHT by the fact that, on PSB meshes, information regarding normal orientations might be compromised. On the other hand, for SCUdb, the performance of St (DCG = 0.712) parallels that of 3DHT noting that 3DHT remains slightly better (DCG = 0.727).

4.2.4. General performance comparison

In this section, we compare the descriptors that we propose (univariate, concatenated, or multivariate) first among
can be computed in less than one second on the average over
4P C( 2 NN performance further. We also note that, on a Pentium
concatenation [Sr,St,Sc] of size 4608 boosts the DCG and
maining ones, albeit their similarity. Furthermore, the triplet
be reporting aspects on the shape not covered by the re-
3584 and 3584, respectively, increase the DCG and NN scores
wise concatenations [Sr,St], [Sr,Sc], and [St,Sc] of size 2048,
variate densities. It is interesting to observe that the pair-
tivariate density estimation, we use concatenation of the 4-
Sc set and their various combinations. Since pairing the fea-
results in higher dimensions (8 or 12) precluding mul-
ff histogram-based descriptors but also proves to be e
pairwise concatenation which is more storage-efficient and
even more time-efficient than [Sr,St,Sc]: [Sr,St] for PSB and
[St,Sc] for SCU.

The density-based framework does not only outperform
histogram-based descriptors but also proves to be effective
as compared to other more general state-of-the-art shape
descriptors. In fact, based on the scores on PSB test set
reported in [5], the [Sr,St,Sc]-descriptor has the highest DCG
score among all other well-known 3D shape descriptors, as
shown in Figure 11. Except for 3DHT [12] and CAH [11],
all the descriptor scores shown in Figure 11 are taken from
[5]. We refer the reader to [5] for brief descriptions and
acronyms of these descriptors. The [Sr,St,Sc]-descriptor has
(a) and SCUdb (b).

Table 9 finally summarizes the experimental results con-
ducted to compare our density-based descriptors with other
histogram-based descriptors. For both databases, PSB and
SCUdb, the [Sr,St,Sc]-descriptor comes at the top in all per-
ff histogram-based descriptors. For both databases, PSB and

Table 9 finally summarizes the experimental results con-
ducted to compare our density-based descriptors with other
histogram-based descriptors. For both databases, PSB and
SCUdb, the [Sr,St,Sc]-descriptor comes at the top in all per-
formance fields. Furthermore, the second place is taken by a
pairwise concatenation which is more storage-efficient and
even more time-efficient than [Sr,St,Sc]: [Sr,St] for PSB and
[St,Sc] for SCU.

5. CONCLUSION

We have proposed a novel methodology to obtain 3D shape
descriptors and evaluated its impact in a retrieval scenario.
We have shown that shape descriptors derived as kernel den-
sity estimates of local surface features prove more advan-
tageous compared to the count-and-accumulate-based his-
togram descriptors. Firstly, one main advantage accrues from
the fact that our descriptors are true probability density func-
tions of geometrical quantities defined over the model sur-
face. Secondly, our surface sampling is not as crude as just
considering triangle barycenters or as profuse as random
sampling, but judiciously chooses the triangle characteristics.
Thirdly and most importantly, the KDE-based approach
deals with multidimensional surface features as easily as with
scalar features. The bandwidth parameters in KDE provide
more gracious control over finite sample-size and dimen-
sionality problems, while with multivariate histograms one
can only adjust the bin widths [13, 14]. The local surface in-
formation brought by multidimensional features proves to be
more discriminating than scalar ones.

The proposed framework applies to 3D objects repre-
sented as triangular meshes but extension to point-cloud
representations is straightforward. Concerning hidden trian-
gles encountered in triangular “soups,” we remark that we
do not try to detect such degeneracies and process them
as any other triangles. They introduce noise in the density
tems and then with various other descriptors existing
in the literature.

In Table 8, we see the competition within the Sr, St, and
Sc set and their various combinations. Since pairing the fea-
tures results in higher dimensions (8 or 12) precluding mul-
tivariate density estimation, we use concatenation of the 4-
variate densities. It is interesting to observe that the pair-
wise concatenations [Sr,St], [Sr,Sc], and [St,Sc] of size 2048,
3584 and 3584, respectively, increase the DCG and NN scores
significantly. We can conclude that each local feature must
be reporting aspects on the shape not covered by the re-
maining ones, albeit their similarity. Furthermore, the triplet
concatenation [Sr,St,Sc] of size 4608 boosts the DCG and
NN performance further. We also note that, on a Pentium
4 PC (2.4 GHz CPU, 2 GB RAM), the [Sr,St,Sc]-descriptor
can be computed in less than one second on the average over
PSB test set meshes, which indicates that our density-based
descriptors are very time-efficient and suitable for practical
online applications.
estimation but not to the extent to alter the density-based descriptor drastically. Furthermore, hidden triangles present in PSB remain in small proportion and SCUdb models are descriptor drastically. Furthermore, hidden triangles present estimation but not to the extent to alter the density-based approach. A side issue is to ren-

tential improvements of decision fusion. For example, several retrievers can operate in parallel and one can consider rank-weighted reordering of the retrieved objects. A second natural avenue of research is in the direction of second-order features. We will tackle the problem of designing second-order features that would serve as natural proxies for curvature-like quantities. Curvature is in fact difficult to work with because of the estimation inaccuracies involved in its computation. Nevertheless, it can be conjectured that the kernel-based approach, thanks to its smoothing behavior, may be useful in deriving curvature-driven 3D shape descriptors. One of our future objectives is thus to arrive at an exhaustive set of first- and second-order features and to discover computational limits of the density-based approach. A side issue is to render the proposed descriptors more effective in discrimination and more efficient in terms of storage size by adequately sampling the local feature domains for target evaluation points. A further question that should be considered is to which extent

| Table 8: DCG and NN scores for the combination of density-based descriptors. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|                 | Sr              | St              | Sc              | [Sr,St]         | [Sr,Sc]         | [St,Sc]         | [Sr,St,Sc]     |
| PSB DCG         | 0.533           | 0.543           | 0.533           | 0.599           | 0.579           | 0.585           | 0.607           |
| PSB NN          | 0.500           | 0.527           | 0.487           | 0.606           | 0.572           | 0.584           | 0.615           |
| SCUdb DCG       | 0.708           | 0.712           | 0.732           | 0.731           | 0.742           | 0.744           | 0.746           |
| SCUdb NN        | 0.745           | 0.754           | 0.733           | 0.788           | 0.776           | 0.774           | 0.786           |

| Table 9: General performances of histogram and density-based descriptors. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Descriptor      | NN              | FT              | ST              | E              | DCG             | NDCG            |
| PSB             |                 |                 |                 |                 |                 |                 |
| [Sr,St,Sc]      | 0.615           | 0.339           | 0.434           | 0.251           | 0.607           | 0.214           |
| [Sr,St]         | 0.606           | 0.333           | 0.423           | 0.245           | 0.599           | 0.199           |
| 3DHT            | 0.588           | 0.311           | 0.396           | 0.230           | 0.577           | 0.154           |
| D2              | 0.363           | 0.168           | 0.245           | 0.145           | 0.448           | −0.103          |
| EGI             | 0.311           | 0.165           | 0.245           | 0.145           | 0.438           | −0.124          |
| CAH             | 0.332           | 0.159           | 0.229           | 0.137           | 0.433           | −0.133          |
| D1              | 0.256           | 0.119           | 0.185           | 0.107           | 0.397           | −0.207          |
| SCUdb            |                 |                 |                 |                 |                 |                 |
| [Sr,St,Sc]      | 0.786           | 0.518           | 0.617           | 0.355           | 0.746           | 0.106           |
| [St,Sc]         | 0.774           | 0.513           | 0.622           | 0.355           | 0.744           | 0.103           |
| 3DHT            | 0.778           | 0.485           | 0.603           | 0.336           | 0.727           | 0.079           |
| CAH             | 0.678           | 0.427           | 0.536           | 0.309           | 0.681           | 0.010           |
| D1              | 0.643           | 0.366           | 0.486           | 0.272           | 0.646           | −0.042          |
| D2              | 0.643           | 0.355           | 0.467           | 0.264           | 0.643           | −0.048          |
| EGI             | 0.489           | 0.252           | 0.349           | 0.203           | 0.535           | −0.207          |

Our framework should be viewed as an application of kernel density estimation [13, 14] with either variable (triangle or mesh levels) or fixed (database level) bandwidth parameters selection [25]. We have also demonstrated that density-based descriptors are much more discriminative in retrieval when the bandwidth parameters are set at database level as compared to mesh or triangle level setting. We think that the database level strategy smooths out individual shape details and emphasizes global shape properties as appropriate for object retrieval and classification tasks; while the other two options, especially the triangle level strategy, result in an overfitting of the feature density and hamper the descriptor's discrimination ability. Furthermore, the computational advantage of density-based descriptors enabled by FGT with a database-dependent bandwidth matrix is very promising for practical online applications.

When combined together, the multivariate density-based 3D shape descriptors introduced in this work outperform the existing histogram-based techniques in the literature. The retrieval competition took place on two databases, PSB and SCUdb, which are fundamentally different in semantic content and mesh quality. In addition, the performance advantage of density-based descriptors over its competitors is not limited to histogram-based ones, as shown in the more general comparison where our [Sr,St,Sc]-descriptor reaches the top position in the category of purely 3D descriptors reported in [5]. As a side remark, based on nearest-neighbor scores of our descriptors, we conjecture that they would also perform well in recognition applications.

In summary, a general framework using KDE has been developed, that covers existing and novel descriptors. Our method enables the use of arbitrary one- or multidimensional surface features for retrieval, recognition, and classification of 3D objects. Future research will concentrate on potential improvements of decision fusion. For example, several retrievers can operate in parallel and one can consider rank-weighted reordering of the retrieved objects. A second natural avenue of research is in the direction of second-order features. We will tackle the problem of designing second-order features that would serve as natural proxies for curvature-like quantities. Curvature is in fact difficult to work with because of the estimation inaccuracies involved in its computation. Nevertheless, it can be conjectured that the kernel-based approach, thanks to its smoothing behavior, may be useful in deriving curvature-driven 3D shape descriptors. One of our future objectives is thus to arrive at an exhaustive set of first- and second-order features and to discover computational limits of the density-based approach. A side issue is to render the proposed descriptors more effective in discrimination and more efficient in terms of storage size by adequately sampling the local feature domains for target evaluation points. A further question that should be considered is to which extent...
the combination of the available features can be exploited, that is, how large the feature dimension of the multivariate densities can be.

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Figure 11: Comparison of 3D shape descriptors on PSB test set. (Except CAH, 3DHT, and our descriptors, DCG values are taken from [5].)


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